REAL-TIME DELAY ESTIMATION BASED ON DELAY HISTORY IN MANY-SERVER SERVICE SYSTEMS WITH TIME-VARYING ARRIVALS SUPPLEMENTARY MATERIAL

by

Rouba Ibrahim and Ward Whitt

IEOR Department
Columbia University
{rei2101, ww2040}@columbia.edu

Abstract from main paper

We develop new improved real-time delay estimators, based on recent customer delay history, in many-server service systems with time-varying arrivals, both with and without customer abandonment. These delay estimators may be used to make delay announcements. We model the arrival process by a nonhomogeneous Poisson process, which has a deterministic time-varying arrival-rate function. Our estimators effectively cope with time-varying arrivals together with non-exponential service-time and abandonment-time distributions, which are often observed in practice. We use computer simulation to verify that our proposed estimators outperform several natural alternatives.

Keywords: Delay Estimation; Delay Announcements; Time-Varying Arrival Rates; Nonstationary Queues; Simulation; Heavy-Traffic.

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Contents

1. Introduction

In this supplement to the main paper, we present additional simulation results. The rest of this supplement is organized as follows. In §2, we present simulation results quantifying the results of the QL, HOL_r, and HOL estimators in the $M_t/M/s$ model with alternative arrival-rate intensity functions. We introduce a new estimator, HOL_s, which approximates the arrival-rate intensity function by the corresponding average over time, $\bar{\lambda}$. We present separate results for HOL_s and show that it is outperformed by all other estimators considered. In §3, we present simulation results for the $M_t/GI/s$ model with alternative service-time distribution functions. In §4, we present tables with point and 95% confidence interval estimates for the $M_t/M/s+GI$ model. Corresponding tables were shown and discussed in §7 of the main paper. In §5, we present simulation results for the $M_t/GI/s+GI$ model with general service and abandonment-time distributions. We present relevant tables of simulation results in §7.

2. Alternative Arrival-Rate Intensity Functions for the $M_t/M/s$ Model

In this section, we present additional simulation results for the $M_t/M/s$ model. For the arrival-rate intensity function, we consider three periodic functions: (i) sinusoidal, (ii) piecewise linear, and (iii) piecewise quadratic. We fix the number of servers $s = 100$, and vary the arrival rate $\bar{\lambda}$, along with the mean service time $E[S]$, to obtain different values of the traffic intensity, ρ. We measure the time-variability of the arrival process relative to the service times. For a fixed period (cycle) length of the arrival-rate function, a relatively small (large) value of $E[S]$ corresponds to slow (high) time-variability in the arrival process, relative to the service times.

2.1. Sinusoidal Arrival Rates

In §4 of the main paper, we presented analytical results for the QL and HOL_r delay estimators in the $M_t/M/s$ model with sinusoidal arrival rates. The corresponding arrival-rate intensity function is given in (4.5). Equation (4.6) yields an approximation for the difference in performance between QL and HOLr, which is particularly accurate for heavily-loaded systems. Here, we test the accuracy of this approximation by computing the *relative percent difference* (RPD), which is the relative error between simulation point estimates of $\text{ASE}(\text{HOL}_r)/\text{ASE}(\text{QL})$, and numerical values given by (4.6).

In addition to QL, HOL, and HOL_r, we consider the HOL_s estimator which approximates the arrival process by a stationary process with rate equal to the average arrival rate, $\bar{\lambda}$.

Tables 1-6 show simulation point and 95% confidence interval estimates for QL, HOL_r, HOL_s, and HOL, for alternative values of the mean service time, $E[S]$. In particular, we let $E[S]$ range from 5 minutes to 12 hours, assuming a daily cycle. Consistent with theory, QL is the most effective estimator, under the MSE criterion. The second best estimator is the HOL_r estimator, which is more accurate than HOL and HOL_s. The difference in performance between HOL_r and HOL is remarkable for small values of $E[S]$ (e.g, $E[S] = 5$ minutes).

Table 1 shows that, with $E[S] = 5$ minutes, the difference in performance between HOL_r and HOL is remarkable: ASE(HOL)/ASE(HOL_r ranges from about 56 for $\rho = 0.9$ to about 125 for $\rho = 0.98$. The HOL_s estimator performs even worse than HOL. The approximation in (4.6) is accurate: The RPD between simulation point estimates for ASE(HOL_r)/ASE(QL), and numerical values given by (4.6) ranges from about 8% for $\rho = 0.9$ to less than 1% for $\rho = 0.98$.

Simulation results for other models are consistent with those reported above, except that the performances of HOL and HOL_r are closer in this case. For example, Table 4 shows that, with $E[S] = 3$ hours, ASE(HOL)/ASE(HOL_r) ranges from about 3 for $\rho = 0.9$ to about 7 for $\rho = 0.98$. Moreover, the approximation in (4.6) is accurate throughout. For example, Table 6 shows that, with $E[S] = 12$ hours, the reported RPD ranges from about 16% for $\rho = 0.9$ to about 4% for $\rho = 0.98$.

2.2. Piecewise Linear Arrival Rates

We now consider a piecewise-linear arrival rate intensity function. In particular, we assume that

$$\lambda(u) = \begin{cases} (0.5 - 2k)\bar{\lambda} + u & \text{if } 2k\bar{\lambda} \le u \le (2k+1)\bar{\lambda}, \\ (2.5 + 2k)\bar{\lambda} - u & \text{if } (2k+1)\bar{\lambda} \le u \le (2k+2)\bar{\lambda}, \end{cases} \tag{2.1}$$

where $k \in \mathbb{Z}$, and $\bar{\lambda}$ is the average arrival rate. This piecewise-linear arrival-rate function periodically increases from $\bar{\lambda}/2$ to $3\bar{\lambda}/2$, and decreases from $3\bar{\lambda}/2$ to $\bar{\lambda}/2$. The period (cycle) length is $2\bar{\lambda}$. The case of piecewise linear arrival rates is interesting because it can serve as an approximation to more complicated arrival-rate intensity functions.

With piecewise linear arrival rates, $\theta_{HOL_r}(t, w)$ can be easily computed using Equation (3.4) of the main paper, together with (2.1) above. We can also compare the performance of QL and HOL_r. In order to compute the integral in (4.4), using (2.1), we would need to know the exact values of $t - w$ and t (since the intensity function is defined piecewise). That is why we approximate $\lambda(u)$ in (2.1) by the average arrival rate $\bar{\lambda}$. By this approximation, the SCV in (4.4) becomes

$$c^2_{W_{HOL}}(t,w) = \frac{2(1+\bar{\lambda}w)}{(2+\bar{\lambda}w)^2} \ . \tag{2.2}$$

Let $W(t)$ be the potential waiting time at time t, the time that an arrival at t would have to wait before beginning service. Since

$$W(t) = \sum_{i=1}^{Q(t)+1} S_i/s \ , \tag{2.3}$$

where $Q(t)$ is the number of customers waiting in queue upon arrival at t, the law of large numbers implies that $W(t)/Q(t) \to 1/s\mu$ as $Q(t) \to \infty$. Using (2.3), and assuming that n is large so that $w \approx n/s\mu$, yields

$$\frac{c^2_{W_{HOL(t,w)}}}{c^2_{W_{QL(n)}}} \approx \frac{2(1+\bar{\lambda}w)(n+1)}{(2+\bar{\lambda}w)^2} \approx \frac{2(n+1)+2\rho n^2+n\rho}{(2+\rho n)^2} \to \frac{2}{\rho} \text{ as } n \to \infty \ , \tag{2.4}$$

which again coincides with (4.6).

Tables 7-12 show simulation point and 95% confidence interval estimates for QL, HOL_r, HOL_s, and HOL, for alternative values of the mean service time, $E[S]$. In particular, we let $E[S]$ range from slightly over 1 minute ($E[S] = 2\bar{\lambda}/1000$) to 12 hours ($E[S] = 2\bar{\lambda}/2$), assuming a daily cycle. Consistent with theory, QL is the most effective estimator, under the MSE criterion. The second best estimator is the HOL_r estimator, which is more accurate than HOL and HOL_s. The difference in performance between HOL_r and HOL is, once more, remarkable for small values of $E[S]$.

Table 7 shows that HOL_r performs, once more, much better than HOL. Indeed, the ratio $\text{ASE}(\text{HOL}_r)/\text{ASE}(\text{HOL})$ ranges from about 86 for $\rho = 0.9$ to about 227 for $\rho = 0.98$. The approximation in (2.4) is, once more, accurate. The RPD reported ranges from about roughly 9% for $\rho = 0.9$ to less than 1% for $\rho = 0.98$. The approximation is more accurate for higher values of ρ, as expected.

Simulation results for other values of $E[S]$ are consistent with the above, with one notable exception. The performances of HOL and HOL_r are now closer than before. For example, Table 10 shows that with $E[S] = 288$ minutes ($E[S] = 2\bar{\lambda}/10$) shows that $\text{ASE}(\text{HOL})/\text{ASE}(\text{HOL}_r)$ ranges from about 2.2 for $\rho = 0.9$ to about 5 for $\rho = 0.98$. As before, the difference in performance between HOL and HOL_r increases as ρ increases. The approximation in (2.4) remains accurate: The RPD reported ranges from about -11% for $\rho = 0.9$ to less than -3% for $\rho = 0.98$. Table 12 shows that, with $E[S] = 12$ hours, the performances of HOL and HOL_r are closer than before. The ratio $\text{ASE}(\text{HOL})/\text{ASE}(\text{HOL}_r)$ ranges from about 1.7 for $\rho = 0.9$, to about 2 for $\rho = 0.98$. The accuracy of the approximation in (2.4) is as before: The RPD reported ranges from about -20% for $\rho = 0.9$ to about -3% for $\rho = 0.98$.

2.3. Piecewise Quadratic Arrival Rates

Finally, we consider a piecewise quadratic arrival-rate intensity function. In particular, we let

$$\lambda(u) = \frac{1}{(2/3)\bar{\lambda}}[-(u - 2k\bar{\lambda})^2 + 2\bar{\lambda}(u - 2k\bar{\lambda})] \quad \text{for} \quad 2k\bar{\lambda} \le u \le 2(k+1)\bar{\lambda} \ , \tag{2.5}$$

where $k \in \mathbb{Z}$. This intensity function periodically increases from 0 to $3\bar{\lambda}/2$ and decreases back to 0; the period length is $2\bar{\lambda}$. Once more, approximating $\lambda(u)$ by $\bar{\lambda}$ leads to

$$\frac{c^2_{W_{HOL(t,w)}}}{c^2_{W_{QL(n)}}} \approx \frac{2(1+\bar{\lambda}w)(n+1)}{(2+\bar{\lambda}w)^2} \approx \frac{2(n+1)+2\rho n^2 + n\rho}{(2+\rho n)^2} \to \frac{2}{\rho} \text{ as } n \to \infty \ , \tag{2.6}$$

which coincides with Equation (4.6) of the main paper.

Tables 13-18 show that simulation results with quadratic arrival rates are consistent with those reported for sinusoidal and linear arrival rates. Yet again, the difference in performance between HOL and HOL_r is extreme for small values of $E[S]$. Table 13 shows that, with $E[S] = 2\bar{\lambda}/1000$, ASE(HOL)/ASE($\text{HOL}_r$) ranges from about 227 for $\rho = 0.9$ to about 530 for $\rho = 0.98$. That is, we see a remarkable improvement in performance due to a simple refinement of the HOL estimator. The approximation in (2.6) is accurate as well: The RPD reported ranges from -12% for $\rho = 0.9$ to about 4% for $\rho = 0.98$. Table 18 shows that, with $E[S] = 12$ hours, ASE(HOL)/ASE(HOL_r) ranges from about 2 for $\rho = 0.9$ to about 4 for $\rho = 0.98$. Moreover, the approximation in (2.6) is accurate: The RPD reported ranges from about -16% for $\rho = 0.9$ to about -4% for $\rho = 0.98$.

3. Results for the $M_t/GI/s$ Model

In this section, we present simulation results quantifying the performance of the estimators in the $M_t/GI/s$ model, with H_2 and $LN(1,1)$ service times. We consider the QL, HOL, HOL_r, and HOL_s estimators. Corresponding results for D, $LN(1,4)$, and M service times appear in the main paper.

For the arrival-rate intensity function, we consider sinusoidal arrivals; see Equation (4.5) of the main paper. For the relative frequency γ, we consider $\gamma = 0.131$ and $\gamma = 1.571$ which correspond to a mean service time $E[S] = 30$ minutes, and $E[S] = 6$ hours respectively. For the relative amplitude α, we consider $\alpha = 0.1$ and $\alpha = 0.5$.

Table 19 shows that, with H_2 service times and $E[S] = 6$ hours, the QL estimator remains the most effective estimator. The RRASE of QL ranges from about 20% for $\rho = 0.9$ to about 13% for $\rho = 0.98$. The HOL_r estimator is the second best estimator, with RRASE(HOL_r) ranging from about 26% for $\rho = 0.9$ to about 16% for $\rho = 0.98$. The difference in performance between QL and HOL_r is less than predicted by (2.6). Indeed, ASE(HOL_r)/ASE(QL) ranges from about 1.7 for $\rho = 0.9$ to about 1.5 for $\rho = 0.98$. The HOL and HOL_s estimators perform nearly the same, and worse than HOL_r. For example, ASE(HOL)/ASE(HOL_r) is roughly equal to 1.7 for $\rho = 0.98$. Thus, we see an improvement in performance due to the refinement of HOL introduced in the paper.

Table 20 shows that, with H_2 service times and $E[S] = 30$ minutes, the QL estimator is, yet again, the most effective estimator. The RRASE of QL ranges from about 20% for $\rho = 0.9$ to about 12% for $\rho = 0.98$. The HOL_r, HOL, and HOL_s estimators perform nearly the same in this model. That is expected, since the arrival-rate intensity function in this model is not highly time-varying. That is, the HOL estimator behaves nearly as with a stationary arrival process. The RRASE of HOL ranges from about 30% for $\rho = 0.9$ to about 17% for $\rho = 0.98$. The difference in performance between HOL and QL is close to that in a stationary system, particularly when the system is heavily loaded. Indeed, ASE(HOL)/ASE(QL) is roughly equal to 2 with $\rho = 0.98$.

Tables 21 and 22 show consistent results with $LN(1,1)$ service times. The QL estimator remains the most effective, and its performance is slightly better than with H_2 service times. That is expected, since the $LN(1,1)$ service-time distribution has the same squared coefficient of variation (SCV) as the exponential distribution (SCV = 1), for which the QL estimator is provably the most accurate estimator, under the MSE criterion. For example, with $E[S] = 30$ minutes, Table 22 shows that RRASE(QL) ranges from about 20% for $\rho = 0.9$ to about 9% for $\rho = 0.98$. The HOL_r estimator is the second most effective estimator. The difference in performance between HOL_r, HOL, and HOL_s is minimal for $E[S] = 30$ minutes; see Table 22. The difference is more significant, however, with $E[S] = 6$ hours. For example, Table 21 shows that ASE(HOL)/ASE(HOL_r) is roughly equal to 3 for $\rho = 0.98$. The accuracy of the approximation in (2.6) is also more accurate than with H_2 service times. Indeed, ASE(HOL_r)/ASE(QL) ranges from about 2 for $\rho = 0.9$ to about 2.2 for $\rho = 0.98$. That is, the RPD between simulation point estimates and numerical values given by (2.6) range from about 9% for $\rho = 0.9$ to about 7% for $\rho = 0.98$.

4. Results for the $M_t/M/s + GI$ Model

In this section, we present tables with point and 95% confidence interval estimates for the $M_t/M/s + GI$ model, presented and discussed in §7 of the main paper. We include results for the QL estimator in those tables. We present results for M, H_2, and E_{10} abandonment in Tables 23, 24, and 25, respectively. For a discussion of the results, the reader is referred to §7 of the main paper.

5. Results for the $M_t/GI/s + GI$ Model

In this section, we present simulation results for the $M_t/GI/s + GI$ model. For the service-time distribution, we consider D and H_2 distributions. For the abandonment-time distribution, we consider M, H_2, and E_{10}. With sinusoidal arrival rates, we consider a relative frequency $\gamma = 1.571$ which corresponds to a mean service time $E[S] = 6$ hours with daily cycles. For the relative amplitude, we let $\alpha = 0.5$. Our simulation results are based on 10 independent replications of length 1 month each.

5.1. D service times

In Tables 26, 28, and 30, we present simulation results for the $M_t/D/s + M$, $M_t/D/s + H_2$, and $M_t/D/s + E_{10}$ models, respectively.

Tables 26 and 28 show that, with both M and H_2 abandonment, we get simulation results consistent those reported earlier.

Table 26 shows that, with M abandonment, QL_m remains the most effective estimator, under the MSE criterion. The RRASE of QL_m ranges from about 17% for $s = 100$ to about 14% for $s = 1000$. The second best estimator is the QL_h estimator. The RRASE of QL_h ranges from about 20% for $s = 100$ to about 15% for $s = 1000$. The difference in performance between QL_m and QL_h is not too great: $\mathrm{ASE}(\mathrm{QL}_h)/\mathrm{ASE}(\mathrm{QL}m)$ ranges from about 1.5 for $s = 100$ to 1.05 for $s = 1000$. That is, the performance of QL_m and QL_h is roughly the same for large values of s. That is to be expected, since both estimators are asymptotically correct. The least effective estimator, among those considered, is the HOL estimator. The RRASE of HOL is close to 30% for all values of s considered. The HOL estimator is not asymptotically correct for this model, as expected.

Table 28 shows that we get similar results with H_2 abandonment. In this case, QL_h is the most effective estimator, under the MSE criterion. The RRASE of QL_h ranges from about

25% for $s = 100$ to about 15% for $s = 1000$. The difference in performance between QL_m and QL_h is not too great: $\text{ASE}(\text{QL}_m)/\text{ASE}(\text{QL}_h)$ ranges from about 0.95 for $s = 100$ to about 1.34 for $s = 1000$. The HOL estimator is, once more, the least effective estimator: $\text{ASE}(\text{HOL})/\text{ASE}(\text{QL}_h)$ ranges from about 2 for $s = 100$ to about 3 for $s = 1000$.

With E_{10} abandonment, Table 30 shows that we get different results than above. Indeed, the performance of all estimators is bad, with performance tending to be independent of the number of servers in the system. The performance of QL_m and QL_h is nearly the same, and they are both somewhat ineffective: $\text{RRASE}(\text{QL}_h)$ is close to 20% for all values of s considered. Moreover, plots of $s\times$ ASE for all estimators show that all estimators are not asymptotically correct in this model. The HOL estimator is, once more, the least effective estimator. The ratio $\text{ASE}(\text{HOL})/\text{ASE}(\text{QL}_h)$ is close to 3 for all values of s considered. There is a need to consider other estimators in this model. We leave this interesting research direction for future work.

5.2. H_2 service times

In Tables 27, 29, and 31, we present simulation results for the $M_t/H_2/s+M$, $M_t/H_2/s+H_2$, and $M_t/H_2/s+E_{10}$ models, respectively.

Table 27 shows that QL_m remains the most effective estimator in this model. The RRASE of QL_m ranges from about 16% for $s = 100$ to less than 5% for $s = 1000$. The QL_h estimator is the second best estimator in this model. The ratio $\text{ASE}(\text{QL}_h)/\text{ASE}(\text{QL}_m)$ is close to 1.5 for all values of s considered. The HOL estimator is, once more, the least effective estimator, among those considered. The RRASE of HOL ranges from about 30% for $s = 100$ to about 24% for $s = 1000$. The ratio $\text{ASE}(\text{HOL})/\text{ASE}(\text{QL}_m)$ ranges from about 4 for $s = 100$ to about 24 for $s = 1000$. Once more, we see a significant degradation in the performance of HOL, with time-varying arrivals.

Table 29 shows that, with H_2 abandonment, QL_m is no longer the most effective estimator, particularly for a large number of servers. The ratio $\text{ASE}(\text{QL}_m)/\text{ASE}(\text{QL}_h)$ ranges from about 0.8 for $s = 100$ to about 2.5 for $s = 1000$. The RRASE of QL_h (which is the best possible in this model) ranges from about 21% for $s = 100$ to about 6% for $s = 1000$. The RRASE of QL_m ranges from about 20% for $s = 100$ to about 11% for $s = 1000$. The HOL estimator is the least effective estimator: $\text{ASE}(\text{HOL})/\text{ASE}(\text{QL}_h)$ ranges from about 2 for $s = 100$ to about 8 for $s = 1000$. Plots of $s\times$ASE of the estimators show that QL_h is asymptotically correct, whereas QL_m and HOL are not.

Table 31 shows that, with E_{10} abandonment, QL_h is the most effective estimator, under the MSE criterion. The QL_m estimator is the second best estimator. The ratio $\mathrm{ASE}(\mathrm{QL}_m)/\mathrm{ASE}(\mathrm{QL}_h)$ ranges from about 1.4 for $s = 100$ to about 7 for $s = 1000$. The RRASE of QL_h (which is the best possible in this model) ranges from about 10% for $s = 100$ to less than 5% for $s = 1000$. The RRASE of QL_m is close to 10% for all s considered. The HOL estimator is the least effective estimator: $\mathrm{ASE}(\mathrm{HOL})/\mathrm{ASE}(\mathrm{QL}_h)$ ranges from about 6 for $s = 100$ to about 34 for $s = 1000$. Plots of $s\times$ASE of the estimators show that QL_h is asymptotically correct, whereas QL_m and HOL are not.

6. Estimating the Required Additional Information for HOL_r

The statistical accuracy of HOL_r is obtained at the expense of ease of implementation. In addition to the HOL delay, w, HOLr depends on the arrival-rate function, $\lambda(t)$, and the mean time between successive service completions (which equals $1/s\mu$ with s simultaneously busy servers and i.i.d. exponential service times with rate μ); see Equation (4.2) of the main paper. In practice, the implementation of HOL_r requires knowledge of those system parameters, which may require estimation from data. Any estimation procedure inevitably produces some estimation error, which would affect the performance of HOL_r.

In this section, we describe additional simulation experiments quantifying the effect of additional information on HOL_r. In particular, we would like to assess the level of error x that is allowed for the performance of HOL_r to remain better than that of HOL. In general, we find that the relative of admissible error x is around 5%; see Tables 32-43. Note that the length of estimation interval needed for each of the service-time distributions depends on the variability of the distribution itself. In particular, high variability distributions such as H_2 require longer intervals.

7. Simulation Results

Efficiency of QL, HOL, HOL_s, and HOL_r with sinusoidal arrival rates

ρ	QL	HOL_r	HOL	HOL_s	HOL_r/QL	(4.6)	RPD (%)
0.9	1.480×10^{-1}	3.020×10^{-1}	16.92	22.15	2.04	2.22	8.20%
	$\pm 6.81 \times 10^{-3}$	$\pm 6.42 \times 10^{-3}$	$\pm 1.36 \times 10^{-1}$	$\pm 1.64 \times 10^{-1}$			
0.93	1.766×10^{-1}	3.511×10^{-1}	28.05	32.67	1.99	2.15	7.56 %
	$\pm 6.05 \times 10^{-3}$	$\pm 8.84 \times 10^{-3}$	$\pm 2.72 \times 10^{-1}$	$\pm 3.09 \times 10^{-1}$			
0.95	2.019×10^{-1}	4.103×10^{-1}	38.06	41.78	2.03	2.11	3.47 %
	$\pm 7.42 \times 10^{-3}$	$\pm 1.79 \times 10^{-2}$	$\pm 3.19 \times 10^{-1}$	$\pm 3.37 \times 10^{-1}$			
0.97	2.159×10^{-1}	4.311×10^{-1}	49.79	52.19	2.00	2.06	3.12%
	$\pm 6.56 \times 10^{-3}$	$\pm 1.40 \times 10^{-2}$	$\pm 4.35 \times 10^{-1}$	$\pm 4.54 \times 10^{-1}$			
0.98	2.261×10^{-1}	4.538×10^{-1}	56.31	57.92	2.01	2.04	1.68%
	$\pm 6.88 \times 10^{-3}$	$\pm 2.34 \times 10^{-2}$	$\pm 3.95 \times 10^{-1}$	$\pm 4.05 \times 10^{-1}$			

Table 1: A comparison of the efficiency of QL, HOL, HOL_r, and HOL_s as a function of the traffic intensity ρ, for sinusoidal arrival rates with relative amplitude $\alpha = 0.5$ and relative frequency $\gamma = 0.022$ (corresponding to a mean service time of 5 minutes). Point estimates of the relative percent difference (RPD) between simulation point estimates and numerical values given by Equation(4.6) are shown.

Efficiency of QL, HOL, HOL_s, and HOL_r with sinusoidal arrival rates

ρ	QL	HOL_r	HOL	HOL_s	HOL_r/QL	(4.6)	RPD (%)
0.9	2.720×10^{-2}	5.432×10^{-2}	5.759×10^{-1}	7.222×10^{-1}	1.99	2.22	10.1%
	$\pm 3.79 \times 10^{-4}$	$\pm 8.92 \times 10^{-4}$	$\pm 4.82 \times 10^{-3}$	$\pm 5.62 \times 10^{-3}$			
0.93	3.150×10^{-2}	6.317×10^{-2}	9.191×10^{-1}	1.043	2.01	2.15	6.76 %
	$\pm 3.77 \times 10^{-4}$	$\pm 8.78 \times 10^{-4}$	$\pm 1.33 \times 10^{-2}$	$\pm 1.47 \times 10^{-2}$			
0.95	3.471×10^{-2}	6.898×10^{-2}	1.208	1.303	1.99	2.11	5.59%
	$\pm 5.37 \times 10^{-4}$	$\pm 8.29 \times 10^{-4}$	$\pm 1.36 \times 10^{-2}$	$\pm 1.44 \times 10^{-2}$			
0.97	3.779×10^{-2}	7.587×10^{-2}	1.574	1.629	2.00	2.06	2.62 %
	$\pm 4.32 \times 10^{-4}$	$\pm 1.46 \times 10^{-3}$	$\pm 2.46 \times 10^{-2}$	$\pm 2.53 \times 10^{-2}$			
0.98	3.998×10^{-2}	7.847×10^{-2}	1.816	1.848	1.96	2.04	3.82%
	$\pm 4.58 \times 10^{-4}$	$\pm 9.90 \times 10^{-4}$	$\pm 2.15 \times 10^{-2}$	$\pm 2.19 \times 10^{-2}$			

Table 2: A comparison of the efficiency of QL, HOL, HOL_r, and HOL_s as a function of the traffic intensity ρ, for sinusoidal arrival rates with relative amplitude $\alpha = 0.5$ and relative frequency $\gamma = 0.131$ (corresponding to a mean service time of 30 minutes). Point estimates of the relative percent difference (RPD) between simulation point estimates and numerical values given by Equation(4.6) are shown.

Efficiency of QL, HOL, HOL_s, and HOL_r with sinusoidal arrival rates

ρ	QL	HOL_r	HOL	HOL_s	HOL_r/QL	(4.6)	RPD (%)
0.9	1.386×10^{-2}	2.746×10^{-2}	1.660×10^{-1}	1.996×10^{-1}	1.98	2.22	10.8%
	$\pm 1.07 \times 10^{-4}$	$\pm 3.75 \times 10^{-4}$	$\pm 1.82 \times 10^{-3}$	$\pm 2.08 \times 10^{-3}$			
0.93	1.620×10^{-2}	3.217×10^{-2}	2.575×10^{-1}	2.849×10^{-1}	1.98	2.15	7.67%
	$\pm 1.73 \times 10^{-4}$	$\pm 4.46 \times 10^{-4}$	$\pm 2.16 \times 10^{-3}$	$\pm 2.34 \times 10^{-3}$			
0.95	1.766×10^{-2}	3.511×10^{-2}	3.345×10^{-1}	3.539×10^{-1}	1.99	2.11	5.54%
	$\pm 2.05 \times 10^{-4}$	$\pm 4.81 \times 10^{-4}$	$\pm 4.99 \times 10^{-3}$	$\pm 5.24 \times 10^{-3}$			
0.97	1.945×10^{-2}	3.852×10^{-2}	4.447×10^{-1}	4.542×10^{-1}	1.98	2.06	3.96%
	$\pm 2.73 \times 10^{-4}$	$\pm 4.49 \times 10^{-4}$	$\pm 4.13 \times 10^{-3}$	$\pm 4.23 \times 10^{-3}$			
0.98	2.109×10^{-2}	4.20×10^{-2}	5.477×10^{-1}	5.517×10^{-1}	1.99	2.04	2.40%
	$\pm 2.87 \times 10^{-4}$	4.48×10^{-4}	$\pm 1.13 \times 10^{-2}$	1.14×10^{-2}			

Table 3: A comparison of the efficiency of QL, HOL, HOL_r, and HOL_s as a function of the traffic intensity ρ, for sinusoidal arrival rates with relative amplitude $\alpha = 0.5$ and relative frequency $\gamma = 0.262$ (corresponding to a mean service time of 1 hour). Point estimates of the relative percent difference (RPD) between simulation point estimates and numerical values given by Equation(4.6) are shown.

Efficiency of QL, HOL, HOL_s, and HOL_r with sinusoidal arrival rates

ρ	QL	HOL_r	HOL	HOL_s	HOL_r/QL	(4.6)	RPD (%)
0.9	4.531×10^{-3}	8.834×10^{-3}	2.508×10^{-2}	2.677×10^{-2}	1.950	2.22	12.3%
	$\pm 4.47 \times 10^{-5}$	$\pm 5.09 \times 10^{-5}$	$\pm 2.92 \times 10^{-4}$	$\pm 3.23 \times 10^{-4}$			
0.93	5.432×10^{-3}	1.062×10^{-2}	3.799×10^{-2}	3.891×10^{-2}	1.956	2.15	9.06%
	$\pm 4.38 \times 10^{-5}$	$\pm 8.41 \times 10^{-5}$	$\pm 5.64 \times 10^{-4}$	5.90×10^{-4}			
0.95	6.274×10^{-3}	1.226×10^{-2}	5.268×10^{-2}	5.280×10^{-2}	1.95	2.11	7.15%
	$\pm 8.59 \times 10^{-5}$	$\pm 1.23 \times 10^{-4}$	$\pm 9.46 \times 10^{-4}$	$\pm 9.68 \times 10^{-4}$			
0.97	7.625×10^{-3}	1.512×10^{-2}	8.350×10^{-2}	8.247×10^{-2}	1.983	2.062	3.84 %
	$\pm 1.03 \times 10^{-4}$	$\pm 1.98 \times 10^{-4}$	$\pm 2.08 \times 10^{-3}$	$\pm 2.089 \times 10^{-3}$			
0.98	9.289×10^{-3}	1.829×10^{-2}	1.230×10^{-1}	1.210×10^{-1}	1.97	2.04	3.51%
	$\pm 1.66974 \times 10^{-4}$	$\pm 3.72 \times 10^{-4}$	$\pm 3.76 \times 10^{-3}$	$\pm 3.74 \times 10^{-3}$			

Table 4: A comparison of the efficiency of QL, HOL, HOL_r, and HOL_s as a function of the traffic intensity ρ, for sinusoidal arrival rates with relative amplitude $\alpha = 0.5$ and relative frequency $\gamma = 0.785$ (corresponding to a mean service time of 3 hours). Point estimates of the relative percent difference (RPD) between simulation point estimates and numerical values given by Equation(4.6) are shown.

Efficiency of QL, HOL, HOL_s, and HOL_r with sinusoidal arrival rates

ρ	QL	HOL_r	HOL	HOL_s	HOL_r/QL	(4.6)	RPD (%)
0.9	2.241×10^{-3}	4.267×10^{-3}	9.011×10^{-3}	8.751×10^{-3}	1.90	2.22	14.3%
	2.31×10^{-5}	$\pm 3.30 \times 10^{-5}$	$\pm 1.46 \times 10^{-4}$	$\pm 1.53 \times 10^{-4}$			
0.93	2.829×10^{-3}	5.452×10^{-3}	1.413×10^{-2}	1.362×10^{-2}	1.93	2.15	10.4%
	$\pm 2.86 \times 10^{-5}$	$\pm 6.35 \times 10^{-5}$	$\pm 2.53 \times 10^{-4}$	$\pm 2.57 \times 10^{-4}$			
0.95	3.495×10^{-3}	6.820×10^{-3}	2.142×10^{-2}	2.064×10^{-2}	1.95	2.11	7.31%
	$\pm 3.31 \times 10^{-5}$	$\pm 7.26 \times 10^{-5}$	$\pm 2.76 \times 10^{-4}$	$\pm 2.78 \times 10^{-4}$			
0.97	4.817×10^{-3}	9.465×10^{-3}	3.903×10^{-2}	3.771×10^{-2}	1.97	2.06	4.69%
	$\pm 1.22 \times 10^{-4}$	$\pm 2.21 \times 10^{-4}$	$\pm 1.46 \times 10^{-3}$	$\pm 1.44 \times 10^{-3}$			
0.98	6.771×10^{-3}	1.332×10^{-2}	6.329×10^{-2}	6.144×10^{-2}	1.97	2.04	3.57%
	$\pm 3.18 \times 10^{-4}$	$\pm 6.17 \times 10^{-4}$	$\pm 3.91 \times 10^{-3}$	$\pm 3.85 \times 10^{-3}$			

Table 5: A comparison of the efficiency of QL, HOL, HOL_r, and HOL_s as a function of the traffic intensity ρ, for sinusoidal arrival rates with relative amplitude $\alpha = 0.5$ and relative frequency $\gamma = 1.571$ (corresponding to a mean service time of 6 hours). Point estimates of the relative percent difference (RPD) between simulation point estimates and numerical values given by Equation(4.6) are shown.

Efficiency of QL, HOL, HOL_s, and HOL_r with sinusoidal arrival rates

ρ	QL	HOL_r	HOL	HOL_s	HOL_r/QL	(4.6)	RPD (%)
0.9	1.374×10^{-3}	2.550×10^{-3}	4.886×10^{-3}	4.451×10^{-3}	1.86	2.22	16.5%
	$\pm 1.53 \times 10^{-5}$	$\pm 2.82 \times 10^{-5}$	$\pm 9.24 \times 10^{-5}$	$\pm 9.16 \times 10^{-5}$			
0.93	1.831×10^{-3}	3.442×10^{-3}	7.878×10^{-3}	7.289×10^{-3}	1.88	2.15	12.6%
	$\pm 2.08 \times 10^{-5}$	$\pm 4.40 \times 10^{-5}$	$\pm 1.18 \times 10^{-4}$	$\pm 1.16 \times 10^{-4}$			
0.95	2.454×10^{-3}	4.711×10^{-3}	1.250×10^{-2}	1.171×10^{-2}	1.92	2.11	8.81%
	$\pm 1.92 \times 10^{-5}$	$\pm 3.78 \times 10^{-5}$	$\pm 1.27 \times 10^{-4}$	$\pm 1.24 \times 10^{-4}$			
0.97	3.770×10^{-3}	7.336×10^{-3}	2.181×10^{-2}	2.074×10^{-2}	1.95	2.06	5.63%
	$\pm 1.27 \times 10^{-4}$	$\pm 2.47 \times 10^{-4}$	$\pm 5.96 \times 10^{-4}$	$\pm 5.84 \times 10^{-4}$			
0.98	5.599×10^{-3}	1.095×10^{-2}	3.043×10^{-2}	2.931×10^{-2}	1.96	2.04	4.16%
	$\pm 2.34 \times 10^{-4}$	$\pm 4.60 \times 10^{-4}$	$\pm 6.46 \times 10^{-4}$	$\pm 6.54 \times 10^{-4}$			

Table 6: A comparison of the efficiency of QL, HOL, HOL_r, and HOL_s as a function of the traffic intensity ρ, for sinusoidal arrival rates with relative amplitude $\alpha = 0.5$ and relative frequency $\gamma = 3.14$ (corresponding to a mean service time of 12 hours). Point estimates of the relative percent difference (RPD) between simulation point estimates and numerical values given by Equation(4.6) are shown.

Efficiency of QL, HOL, HOL_s, and HOL_r with linear arrival rates

ρ	QL	HOL_r	HOL	HOL_s	HOL_r/QL	(2.4)	RPD (%)
0.9	6.458×10^{-2} $\pm 3.95 \times 10^{-3}$	1.307×10^{-1} $\pm 5.48 \times 10^{-3}$	11.32 $\pm 1.54 \times 10^{-1}$	16.01 $\pm 1.97 \times 10^{-1}$	2.02	2.22	-8.95%
0.93	8.440×10^{-2} $\pm 4.13 \times 10^{-3}$	1.687×10^{-1} $\pm 6.74 \times 10^{-3}$	22.16 $\pm 2.65 \times 10^{-1}$	26.72 $\pm 3.08 \times 10^{-1}$	2.00	2.15	-7.07%
0.95	1.008×10^{-1} $\pm 5.09 \times 10^{-3}$	1.937×10^{-1} $\pm 9.16 \times 10^{-3}$	32.53 $\pm 2.40 \times 10^{-1}$	36.29 $\pm 2.62 \times 10^{-1}$	1.92	2.11	-8.76%
0.97	1.112×10^{-1} $\pm 4.84 \times 10^{-3}$	2.289×10^{-1} $\pm 7.758 \times 10^{-3}$	46.32 $\pm 4.54 \times 10^{-1}$	48.84 $\pm 4.74 \times 10^{-1}$	2.06	2.06	-0.151%
0.98	1.168×10^{-1} $\pm 4.19 \times 10^{-3}$	2.407×10^{-1} $\pm 7.60 \times 10^{-3}$	54.71 $\pm 4.90 \times 10^{-1}$	56.45 $\pm 5.10 \times 10^{-1}$	2.06	2.04	0.959%

Table 7: A comparison of the efficiency of QL, HOL, HOL_r, and HOL_s as a function of the traffic intensity ρ, for piecewise linear arrival rates with $\bar{\lambda}$ and μ corresponding to a mean service time of 1/1000 times the period length. Point estimates of the relative percent difference (RPD) between simulation point estimates and numerical values given by Equation (2.4) are shown.

Efficiency of QL, HOL, HOL_s, and HOL_r with linear arrival rates

ρ	QL	HOL_r	HOL	HOL_s	HOL_r/QL	(2.4)	RPD (%)
0.9	7.605×10^{-2} $\pm 5.86 \times 10^{-4}$	1.507×10^{-1} $\pm 6.14 \times 10^{-4}$	7.396×10^{-1} $\pm 5.93 \times 10^{-3}$	9.324×10^{-1} $\pm 7.20 \times 10^{-3}$	1.98	2.22	-10.8%
0.93	9.529×10^{-2} $\pm 1.02 \times 10^{-3}$	1.885×10^{-1} $\pm 2.17 \times 10^{-3}$	1.274 $\pm 1.27 \times 10^{-2}$	1.443 $\pm 1.40 \times 10^{-2}$	1.98	2.15	-8.00%
0.95	1.071×10^{-1} $\pm 1.14 \times 10^{-3}$	2.153×10^{-1} $\pm 1.74 \times 10^{-3}$	1.753 $\pm 2.01 \times 10^{-2}$	1.880 2.15×10^{-2}	2.01	2.11	-4.50%
0.97	1.209×10^{-1} $\pm 1.40 \times 10^{-3}$	2.418×10^{-1} $\pm 2.69 \times 10^{-3}$	2.429 $\pm 2.13 \times 10^{-2}$	2.499 $\pm 2.20 \times 10^{-2}$	2.00	2.06	-3.00%
0.98	1.317×10^{-1} $\pm 1.06 \times 10^{-3}$	2.634×10^{-1} $\pm 2.30 \times 10^{-3}$	2.977 $\pm 4.82 \times 10^{-2}$	3.014 $\pm 4.89 \times 10^{-2}$	2.000	2.041	-2.00%

Table 8: A comparison of the efficiency of QL, HOL, HOL_r, and HOL_s as a function of the traffic intensity ρ, for piecewise linear arrival rates with $\bar{\lambda}$ and μ corresponding to a mean service time of 1/40 times the period length. Point estimates of the relative percent difference (RPD) between simulation point estimates and numerical values given by Equation (2.4) are shown.

Efficiency of QL, HOL, HOL_s, and HOL_r with linear arrival rates

ρ	QL	HOL_r	HOL	HOL_s	HOL_r/QL	(2.4)	RPD (%)
0.9	7.884×10^{-2}	1.559×10^{-1}	4.934×10^{-1}	5.786×10^{-1}	1.98	2.22	−11.0%
	$\pm 4.69 \times 10^{-4}$	$\pm 9.61 \times 10^{-4}$	3.09×10^{-3}	$\pm 3.56 \times 10^{-3}$			
0.93	9.732×10^{-2}	1.940×10^{-1}	7.900×10^{-1}	8.577×10^{-1}	1.99	2.15	−7.32%
	$\pm 9.88 \times 10^{-4}$	$\pm 2.28 \times 10^{-3}$	$\pm 1.12 \times 10^{-2}$	$\pm 1.23 \times 10^{-2}$			
0.95	1.117×10^{-1}	2.207×10^{-1}	1.074	1.119	1.98	2.11	−6.13%
	$\pm 6.33 \times 10^{-4}$	$\pm 9.88 \times 10^{-4}$	$\pm 8.63 \times 10^{-3}$	$\pm 9.05 \times 10^{-3}$			
0.97	1.305×10^{-1}	2.573×10^{-1}	1.552	1.567	1.97	2.06	−4.36%
	$\pm 1.04 \times 10^{-3}$	$\pm 2.02 \times 10^{-3}$	$\pm 1.98 \times 10^{-2}$	$\pm 2.02 \times 10^{-2}$			
0.98	1.481×10^{-1}	2.942×10^{-1}	2.099	2.098	1.99	2.04	−2.62%
	$\pm 2.03 \times 10^{-3}$	$\pm 5.05 \times 10^{-3}$	$\pm 5.77 \times 10^{-2}$	$\pm 5.82 \times 10^{-2}$			

Table 9: A comparison of the efficiency of QL, HOL, HOL_r, and HOL_s as a function of the traffic intensity ρ, for piecewise linear arrival rates with $\bar{\lambda}$ and μ corresponding to a mean service time of 1/20 times the period length. Point estimates of the relative percent difference (RPD) between simulation point estimates and numerical values given by Equation (2.4) are shown.

Efficiency of QL, HOL, HOL_s, and HOL_r with linear arrival rates

ρ	QL	HOL_r	HOL	HOL_s	HOL_r/QL	(2.4)	RPD (%)
0.9	8.021×10^{-2}	1.573×10^{-1}	3.506×10^{-1}	3.746×10^{-1}	1.96	2.22	−11.8%
	$\pm 6.17 \times 10^{-4}$	$\pm 1.29 \times 10^{-3}$	$\pm 3.85 \times 10^{-3}$	$\pm 4.19 \times 10^{-3}$			
0.93	1.005×10^{-1}	1.966×10^{-1}	5.400×10^{-1}	5.533×10^{-1}	1.96	2.15	−9.00%
	$\pm 5.58 \times 10^{-4}$	$\pm 1.50 \times 10^{-3}$	$\pm 7.33 \times 10^{-3}$	$\pm 7.81 \times 10^{-3}$			
0.95	1.184×10^{-1}	2.330×10^{-1}	7.535×10^{-1}	7.551×10^{-1}	1.97	2.11	−6.56%
	$\pm 4.27 \times 10^{-4}$	$\pm 1.62 \times 10^{-3}$	$\pm 6.86 \times 10^{-3}$	$\pm 7.06 \times 10^{-3}$			
0.97	1.486×10^{-1}	2.935×10^{-1}	1.218	1.204	1.97	2.06	−4.24%
	$\pm 1.36 \times 10^{-3}$	$\pm 3.01 \times 10^{-3}$	$\pm 2.08 \times 10^{-2}$	$\pm 2.10 \times 10^{-2}$			
0.98	1.798×10^{-1}	3.566×10^{-1}	1.815	1.789	1.98	2.04	−2.85%
	$\pm 4.46 \times 10^{-3}$	$\pm 9.59 \times 10^{-3}$	$\pm 8.95 \times 10^{-2}$	$\pm 8.97 \times 10^{-2}$			

Table 10: A comparison of the efficiency of QL, HOL, HOL_r, and HOL_s as a function of the traffic intensity ρ, for piecewise linear arrival rates with $\bar{\lambda}$ and μ corresponding to a mean service time of 1/10 times the period length. Point estimates of the relative percent difference (RPD) between simulation point estimates and numerical values given by Equation (2.4) are shown.

Efficiency of QL, HOL, HOL_s, and HOL_r with linear arrival rates

ρ	QL	HOL_r	HOL	HOL_s	HOL_r/QL	(2.4)	RPD (%)
0.9	8.215×10^{-2}	1.576×10^{-1}	2.820×10^{-1}	2.746×10^{-1}	1.92	2.22	−13.7%
	$\pm 4.75 \times 10^{-4}$	$\pm 1.05 \times 10^{-3}$	$\pm 2.60 \times 10^{-3}$	$\pm 2.74 \times 10^{-3}$			
0.93	1.074×10^{-1}	2.067×10^{-1}	4.400×10^{-1}	4.255×10^{-1}	1.93	2.15	−10.5%
	$\pm 3.95 \times 10^{-4}$	$\pm 1.00 \times 10^{-3}$	$\pm 4.35 \times 10^{-3}$	$\pm 4.51 \times 10^{-3}$			
0.95	1.350×10^{-1}	2.624×10^{-1}	6.646×10^{-1}	6.422×10^{-1}	1.94	2.11	−7.68%
	$\pm 1.19 \times 10^{-3}$	$\pm 2.53 \times 10^{-3}$	$\pm 1.19 \times 10^{-2}$	$\pm 1.21 \times 10^{-2}$			
0.97	1.873×10^{-1}	3.670×10^{-1}	1.191	1.154	1.96	2.06	−4.97%
	$\pm 1.56 \times 10^{-3}$	$\pm 2.83 \times 10^{-3}$	$\pm 1.53 \times 10^{-2}$	$\pm 1.52 \times 10^{-2}$			
0.98	2.610×10^{-1}	5.137×10^{-1}	2.024	1.972	1.97	2.04	−3.56%
	$\pm 8.57 \times 10^{-3}$	$\pm 1.64 \times 10^{-2}$	$\pm 1.05 \times 10^{-1}$	1.04×10^{-1}			

Table 11: A comparison of the efficiency of QL, HOL, HOL_r, and HOL_s as a function of the traffic intensity ρ, for piecewise linear arrival rates with $\bar{\lambda}$ and μ corresponding to a mean service time of 1/5 times the period length. Point estimates of the relative percent difference (RPD) between simulation point estimates and numerical values given by Equation (2.4) are shown.

Efficiency of QL, HOL, HOL_s, and HOL_r with linear arrival rates

ρ	QL	HOL_r	HOL	HOL_s	HOL_r/QL	(2.4)	RPD (%)
0.9	1.144×10^{-1}	2.105×10^{-1}	3.510×10^{-1}	3.184×10^{-1}	1.84	2.22	−17.2%
	$\pm 1.09 \times 10^{-3}$	$\pm 2.04 \times 10^{-3}$	$\pm 4.44 \times 10^{-3}$	$\pm 4.48 \times 10^{-3}$			
0.93	1.586×10^{-1}	2.990×10^{-1}	5.663×10^{-1}	5.236×10^{-1}	1.89	2.15	−12.3%
	$\pm 1.49 \times 10^{-3}$	$\pm 2.83 \times 10^{-3}$	$\pm 6.76 \times 10^{-3}$	$\pm 6.78 \times 10^{-3}$			
0.95	2.183×10^{-1}	4.181×10^{-1}	8.859×10^{-1}	8.311×10^{-1}	1.92	2.11	−9.02%
	$\pm 2.52 \times 10^{-3}$	$\pm 6.49 \times 10^{-3}$	$\pm 1.57 \times 10^{-2}$	$\pm 1.54 \times 10^{-2}$			
0.97	3.452×10^{-1}	6.717×10^{-1}	1.546	1.473	1.946	2.062	−5.61%
	$\pm 1.01 \times 10^{-2}$	$\pm 2.06 \times 10^{-2}$	$\pm 3.36 \times 10^{-2}$	$\pm 3.34 \times 10^{-2}$			
0.98	5.281×10^{-1}	1.040	2.269	2.191	1.97	2.04	−3.53%
	$\pm 1.60 \times 10^{-2}$	$\pm 3.06 \times 10^{-2}$	$\pm 4.22 \times 10^{-2}$	$\pm 4.21 \times 10^{-2}$			

Table 12: A comparison of the efficiency of QL, HOL, HOL_r, and HOL_s as a function of the traffic intensity ρ, for piecewise linear arrival rates with $\bar{\lambda}$ and μ corresponding to a mean service time of 1/2 times the period length. Point estimates of the relative percent difference (RPD) between simulation point estimates and numerical values given by Equation (2.4) are shown.

Efficiency of QL, HOL, HOL_s, and HOL_r with quadratic arrival rates

ρ	QL	HOL_r	HOL	HOL_s	HOL_r/QL	(2.6)	RPD (%)
0.9	1.193×10^{-1}	2.326×10^{-1}	52.80	77.45	1.95	2.22	−12.3%
	$\pm 8.93 \times 10^{-3}$	$\pm 1.47 \times 10^{-2}$	$\pm 7.03 \times 10^{-1}$	$\pm 9.89 \times 10^{-1}$			
0.93	1.399×10^{-1}	2.840×10^{-1}	86.83	113.4	2.03	2.15	−5.61%
	$\pm 8.48 \times 10^{-3}$	$\pm 1.99 \times 10^{-2}$	±1.35	±1.65			
0.95	1.475×10^{-1}	2.942×10^{-1}	122.9	146.5	1.99	2.11	−5.26%
	$\pm 1.38 \times 10^{-2}$	$\pm 2.96 \times 10^{-2}$	±1.21	±1.38			
0.97	1.884×10^{-1}	3.421×10^{-1}	171.9	188.6	1.82	2.06	−11.9%
	$\pm 1.85 \times 10^{-2}$	$\pm 2.61 \times 10^{-2}$	±2.36	±2.49			
0.98	1.796×10^{-1}	3.821×10^{-1}	202.5	214.2	2.13	2.04	4.28%
	$\pm 2.07 \times 10^{-2}$	$\pm 3.24 \times 10^{-2}$	±2.69	±2.76			

Table 13: A comparison of the efficiency of QL, HOL, HOL_r, and HOL_s as a function of the traffic intensity ρ, for piecewise linear arrival rates with $\bar{\lambda}$ and μ corresponding to a mean service time of 1/1000 times the period length. Point estimates of the relative percent difference (RPD) between simulation point estimates and numerical values given by Equation (2.6) are shown.

Efficiency of QL, HOL, HOL_s, and HOL_r with quadratic arrival rates

ρ	QL	HOL_r	HOL	HOL_s	HOL_r/QL	(2.6)	RPD (%)
0.9	1.269×10^{-1}	2.528×10^{-1}	2.635	3.654	1.99	2.22	−10.4%
	$\pm 1.75 \times 10^{-3}$	$\pm 2.89 \times 10^{-3}$	$\pm 2.29 \times 10^{-2}$	$\pm 3.21 \times 10^{-2}$			
0.93	1.535×10^{-1}	3.065×10^{-1}	4.292	5.324	2.00	2.15	−7.14%
	$\pm 2.77 \times 10^{-3}$	$\pm 3.56 \times 10^{-3}$	$\pm 5.02 \times 10^{-2}$	$\pm 5.95 \times 10^{-2}$			
0.95	1.702×10^{-1}	3.386×10^{-1}	5.917	6.785	1.99	2.11	−5.49%
	$\pm 2.33 \times 10^{-3}$	$\pm 3.53 \times 10^{-3}$	$\pm 5.57 \times 10^{-2}$	$\pm 6.07 \times 10^{-2}$			
0.97	1.861×10^{-1}	3.676×10^{-1}	8.177	8.735	1.98	2.06	−4.18%
	$\pm 2.84 \times 10^{-3}$	$\pm 5.55 \times 10^{-3}$	$\pm 9.46 \times 10^{-2}$	$\pm 9.85 \times 10^{-2}$			
0.98	1.998×10^{-1}	3.904×10^{-1}	10.05	10.40	1.96	2.04	−4.22%
	$\pm 3.87 \times 10^{-3}$	$\pm 7.93 \times 10^{-3}$	$\pm 1.66 \times 10^{-1}$	$\pm 1.69 \times 10^{-1}$			

Table 14: A comparison of the efficiency of QL, HOL, HOL_r, and HOL_s as a function of the traffic intensity ρ, for piecewise linear arrival rates with $\bar{\lambda}$ and μ corresponding to a mean service time of 1/40 times the period length. Point estimates of the relative percent difference (RPD) between simulation point estimates and numerical values given by Equation (2.6) are shown.

Efficiency of QL, HOL, HOL_s, and HOL_r with quadratic arrival rates

ρ	QL	HOL_r	HOL	HOL_s	HOL_r/QL	(2.6)	RPD (%)
0.9	1.294×10^{-1}	2.569×10^{-1}	1.520	1.996	1.99	2.22	−10.6%
	$\pm 1.45 \times 10^{-3}$	$\pm 2.59 \times 10^{-3}$	$\pm 1.51 \times 10^{-2}$	$\pm 1.86 \times 10^{-2}$			
0.93	1.550×10^{-1}	3.070×10^{-1}	2.406	2.862	1.980	2.151	−7.93%
	$\pm 1.26 \times 10^{-3}$	$\pm 3.35 \times 10^{-3}$	$\pm 3.32 \times 10^{-2}$	$\pm 3.85 \times 10^{-2}$			
0.95	1.707×10^{-1}	3.388×10^{-1}	3.300	3.667	1.98	2.11	−5.72%
	$\pm 1.60 \times 10^{-3}$	$\pm 3.55 \times 10^{-3}$	$\pm 4.97 \times 10^{-2}$	$\pm 5.35 \times 10^{-2}$			
0.97	1.917×10^{-1}	3.847×10^{-1}	4.784	5.000	2.01	2.06	−2.68%
	$\pm 2.15 \times 10^{-3}$	$\pm 4.40 \times 10^{-3}$	$\pm 9.11 \times 10^{-2}$	$\pm 9.33 \times 10^{-2}$			
0.98	2.111×10^{-1}	4.230×10^{-1}	6.288	6.397	2.00	2.04	−1.81%
	$\pm 2.54 \times 10^{-3}$	$\pm 5.80 \times 10^{-3}$	$\pm 1.56 \times 10^{-1}$	$\pm 1.58 \times 10^{-1}$			

Table 15: A comparison of the efficiency of QL, HOL, HOL_r, and HOL_s as a function of the traffic intensity ρ, for piecewise linear arrival rates with $\bar{\lambda}$ and μ corresponding to a mean service time of 1/20 times the period length. Point estimates of the relative percent difference (RPD) between simulation point estimates and numerical values given by Equation (2.6) are shown.

Efficiency of QL, HOL, HOL_s, and HOL_r with quadratic arrival rates

ρ	QL	HOL_r	HOL	HOL_s	HOL_r/QL	(2.6)	RPD (%)
0.9	1.253×10^{-1}	2.471×10^{-1}	8.864×10^{-1}	1.066	1.97	2.22	−11.3%
	$\pm 9.96 \times 10^{-4}$	$\pm 2.79 \times 10^{-3}$	$\pm 9.45 \times 10^{-3}$	$\pm 1.12 \times 10^{-2}$			
0.93	1.536×10^{-1}	3.032×10^{-1}	1.417	1.578	1.97	2.15	−8.24%
	$\pm 1.01 \times 10^{-3}$	$\pm 2.24 \times 10^{-3}$	$\pm 1.68 \times 10^{-2}$	$\pm 1.85 \times 10^{-2}$			
0.95	1.745×10^{-1}	3.454×10^{-1}	2.006	2.120	1.97	2.11	−6.00%
	$\pm 1.24 \times 10^{-3}$	$\pm 2.53 \times 10^{-3}$	$\pm 1.83 \times 10^{-2}$	$\pm 1.91 \times 10^{-2}$			
0.97	2.067×10^{-1}	4.079×10^{-1}	3.227	3.261	1.97	2.06	−4.30%
	$\pm 3.31 \times 10^{-3}$	$\pm 3.97 \times 10^{-3}$	$\pm 1.086 \times 10^{-1}$	$\pm 1.09 \times 10^{-1}$			
0.98	2.433×10^{-1}	4.829×10^{-1}	4.959	4.931	1.98	2.04	−2.74%
	$\pm 7.45 \times 10^{-3}$	$\pm 1.42 \times 10^{-2}$	$\pm 3.22 \times 10^{-1}$	$\pm 3.20 \times 10^{-1}$			

Table 16: A comparison of the efficiency of QL, HOL, HOL_r, and HOL_s as a function of the traffic intensity ρ, for piecewise linear arrival rates with $\bar{\lambda}$ and μ corresponding to a mean service time of 1/10 times the period length. Point estimates of the relative percent difference (RPD) between simulation point estimates and numerical values given by Equation (2.6) are shown.

ρ	QL	HOL_r	HOL	HOL_s	HOL_r/QL	(2.6)	RPD (%)
0.9	1.202×10^{-1} $\pm 7.46 \times 10^{-4}$	2.343×10^{-1} $\pm 2.26 \times 10^{-3}$	5.797×10^{-1} $\pm 8.09 \times 10^{-3}$	6.167×10^{-1} $\pm 8.70 \times 10^{-3}$	1.95	2.22	−12.3%
0.93	1.533×10^{-1} $\pm 1.31 \times 10^{-3}$	2.978×10^{-1} $\pm 2.25 \times 10^{-3}$	9.579×10^{-1} $\pm 1.34 \times 10^{-2}$	9.770×10^{-1} $\pm 1.40 \times 10^{-2}$	1.94	2.15	−9.68%
0.95	1.827×10^{-1} $\pm 2.97 \times 10^{-3}$	3.550×10^{-1} $\pm 5.10 \times 10^{-3}$	1.439 $\pm 4.16 \times 10^{-2}$	1.428 $\pm 4.22 \times 10^{-2}$	1.94	2.11	−7.71%
0.97	2.387×10^{-1} $\pm 5.98 \times 10^{-3}$	4.703×10^{-1} $\pm 1.07 \times 10^{-2}$	2.758 $\pm 1.25 \times 10^{-1}$	2.692 $\pm 1.24 \times 10^{-1}$	1.97	2.06	−4.43%
0.98	3.101×10^{-1} $\pm 1.41 \times 10^{-2}$	6.131×10^{-1} $\pm 2.71 \times 10^{-2}$	4.599 $\pm 3.17 \times 10^{-1}$	4.484 $\pm 3.13 \times 10^{-1}$	1.98	2.04	−3.13%

Efficiency of QL, HOL, HOL_s, and HOL_r with quadratic arrival rates

Table 17: A comparison of the efficiency of QL, HOL, HOL_r, and HOL_s as a function of the traffic intensity ρ, for piecewise linear arrival rates with $\bar{\lambda}$ and μ corresponding to a mean service time of 1/5 times the period length. Point estimates of the relative percent difference (RPD) between simulation point estimates and numerical values given by Equation (2.6) are shown.

Efficiency of QL, HOL, HOL_s, and HOL_r with quadratic arrival rates

ρ	QL	HOL_r	HOL	HOL_s	HOL_r/QL	(2.6)	RPD (%)
0.9	1.354×10^{-1} $\pm 1.08 \times 10^{-3}$	2.528×10^{-1} $\pm 2.15 \times 10^{-3}$	5.290×10^{-1} $\pm 1.03 \times 10^{-2}$	4.877×10^{-1} $\pm 1.04 \times 10^{-2}$	1.87	2.22	−16.0%
0.93	1.845×10^{-1} $\pm 2.44 \times 10^{-3}$	3.509×10^{-1} $\pm 3.66 \times 10^{-3}$	9.475×10^{-1} $\pm 2.39 \times 10^{-2}$	8.787×10^{-1} $\pm 2.37 \times 10^{-2}$	1.90	2.15	−11.6%
0.95	2.468×10^{-1} $\pm 2.86 \times 10^{-3}$	4.732×10^{-1} $\pm 5.09 \times 10^{-3}$	1.554 $\pm 2.76 \times 10^{-2}$	1.451 $\pm 2.68 \times 10^{-2}$	1.92	2.11	−8.93%
0.97	3.767×10^{-1} $\pm 9.22 \times 10^{-3}$	7.348×10^{-1} $\pm 1.81 \times 10^{-2}$	2.797 $\pm 4.84 \times 10^{-2}$	2.650 $\pm 4.73 \times 10^{-2}$	1.95	2.06	−5.38%
0.98	5.419×10^{-1} $\pm 1.70 \times 10^{-2}$	1.065 $\pm 3.47 \times 10^{-2}$	3.958 $\pm 9.69 \times 10^{-2}$	3.799 $\pm 9.63 \times 10^{-2}$	1.97	2.04	−3.71%

Table 18: A comparison of the efficiency of QL, HOL, HOL_r, and HOL_s as a function of the traffic intensity ρ, for piecewise linear arrival rates with $\bar{\lambda}$ and μ corresponding to a mean service time of 1/2 times the period length. Point estimates of the relative percent difference (RPD) between simulation point estimates and numerical values given by Equation (2.6) are shown.

Efficiency of QL, HOL, HOL_s, and HOL_r in the $M_t/H_2/s$ Model with $E[S] = 6$ hours				
ρ	QL	HOL_r	HOL	HOL_s
0.9	2.432×10^{-3}	4.155×10^{-3}	9.021×10^{-3}	8.666×10^{-3}
	$\pm 1.24 \times 10^{-4}$	$\pm 2.07 \times 10^{-4}$	$\pm 3.32 \times 10^{-4}$	$\pm 3.53 \times 10^{-4}$
0.93	3.924×10^{-3}	6.571×10^{-3}	1.572×10^{-2}	1.507×10^{-2}
	$\pm 1.30 \times 10^{-4}$	$\pm 1.96 \times 10^{-4}$	$\pm 4.37 \times 10^{-4}$	$\pm 4.30 \times 10^{-4}$
0.95	6.309×10^{-3}	1.034×10^{-2}	2.436×10^{-2}	2.346×10^{-2}
	$\pm 3.05 \times 10^{-4}$	$\pm 4.55 \times 10^{-4}$	$\pm 6.06 \times 10^{-4}$	$\pm 6.16 \times 10^{-4}$
0.97	1.211×10^{-2}	1.885×10^{-2}	3.767×10^{-2}	3.672×10^{-2}
	$\pm 9.32 \times 10^{-4}$	$\pm 1.42 \times 10^{-3}$	$\pm 1.68 \times 10^{-3}$	$\pm 1.75 \times 10^{-3}$
0.98	2.219×10^{-2}	3.344×10^{-2}	5.536×10^{-2}	5.456×10^{-2}
	$\pm 2.89 \times 10^{-3}$	$\pm 3.90 \times 10^{-3}$	$\pm 3.84 \times 10^{-3}$	$\pm 3.99 \times 10^{-3}$

Table 19: A comparison of the efficiency of QL, HOL, HOL_r, and HOL_s as a function of the traffic intensity ρ, for sinusoidal arrival rates with $\bar{\lambda}$ and μ corresponding to a mean service time of 6 hours. Point and 95% confidence interval estimates of the ASE's are shown.

Efficiency of QL, HOL, HOL_s, and HOL_r in the $M_t/H_2/s$ Model with $E[S] = 30$ minutes				
ρ	QL	HOL_r	HOL	HOL_s
0.9	4.917×10^{-3}	8.058×10^{-3}	8.649×10^{-3}	9.540×10^{-3}
	$\pm 2.38 \times 10^{-4}$	$\pm 3.97 \times 10^{-4}$	$\pm 4.52 \times 10^{-4}$	$\pm 5.55 \times 10^{-4}$
0.93	7.727×10^{-3}	1.243×10^{-2}	1.410×10^{-2}	1.492×10^{-2}
	$\pm 2.55 \times 10^{-4}$	$\pm 3.55 \times 10^{-4}$	$\pm 4.09 \times 10^{-4}$	$\pm 4.53 \times 10^{-4}$
0.95	1.198×10^{-2}	1.865×10^{-2}	2.226×10^{-2}	2.295×10^{-2}
	$\pm 6.50 \times 10^{-4}$	$\pm 7.45 \times 10^{-4}$	$\pm 9.73 \times 10^{-4}$	$\pm 1.06 \times 10^{-3}$
0.97	1.792×10^{-2}	2.766×10^{-2}	3.527×10^{-2}	3.545×10^{-2}
	$\pm 1.25 \times 10^{-3}$	$\pm 1.68 \times 10^{-3}$	$\pm 2.28 \times 10^{-3}$	$\pm 2.37 \times 10^{-3}$
0.98	3.033×10^{-2}	4.461×10^{-2}	6.237×10^{-2}	6.251×10^{-2}
	$\pm 2.33 \times 10^{-3}$	$\pm 3.18 \times 10^{-3}$	$\pm 5.83 \times 10^{-3}$	$\pm 5.97 \times 10^{-3}$

Table 20: A comparison of the efficiency of QL, HOL, HOL_r, and HOL_s as a function of the traffic intensity ρ, for sinusoidal arrival rates with $\bar{\lambda}$ and μ corresponding to a mean service time of 6 hours. Point and 95% confidence interval estimates of the ASE's are shown.

Efficiency of QL, HOL, HOL_s, and HOL_r in the $M_t/LN(1,1)/s$ Model with $E[S] = 6$ hours

ρ	QL	HOL_r	HOL	HOL_s
0.9	1.145×10^{-3}	2.302×10^{-3}	4.472×10^{-3}	4.043×10^{-3}
	$\pm 9.62 \times 10^{-6}$	$\pm 2.06 \times 10^{-5}$	$\pm 6.27 \times 10^{-5}$	$\pm 6.12 \times 10^{-5}$
0.93	1.469×10^{-3}	3.049×10^{-3}	7.152×10^{-3}	6.583×10^{-3}
	$\pm 1.46 \times 10^{-5}$	$\pm 3.10 \times 10^{-5}$	$\pm 1.05 \times 10^{-4}$	$\pm 1.02 \times 10^{-4}$
0.95	1.874×10^{-3}	4.016×10^{-3}	1.105×10^{-2}	1.030×10^{-2}
	$\pm 2.53 \times 10^{-5}$	$\pm 4.65 \times 10^{-5}$	$\pm 2.16 \times 10^{-4}$	$\pm 2.11 \times 10^{-4}$
0.97	2.903×10^{-3}	6.386×10^{-3}	1.962×10^{-2}	1.861×10^{-2}
	$\pm 7.86 \times 10^{-5}$	$\pm 1.92 \times 10^{-4}$	$\pm 4.50 \times 10^{-4}$	$\pm 4.45 \times 10^{-4}$
0.98	4.191×10^{-3}	9.264×10^{-3}	2.788×10^{-2}	2.680×10^{-2}
	$\pm 2.27 \times 10^{-4}$	$\pm 4.40 \times 10^{-4}$	$\pm 6.89 \times 10^{-4}$	$\pm 6.88 \times 10^{-4}$

Table 21: A comparison of the efficiency of QL, HOL, HOL_r, and HOL_s as a function of the traffic intensity ρ, for sinusoidal arrival rates with $\bar{\lambda}$ and μ corresponding to a mean service time of 6 hours. Point and 95% confidence interval estimates of the ASE's are shown.

Efficiency of QL, HOL, HOL_s, and HOL_r in the $M_t/LN(1,1)/s$ Model with $E[S] = 30$ minutes

ρ	QL	HOL_r	HOL	HOL_s
0.9	1.774×10^{-3}	3.761×10^{-3}	4.055×10^{-3}	4.149×10^{-3}
	$\pm 2.76 \times 10^{-5}$	$\pm 4.65 \times 10^{-5}$	$\pm 4.70 \times 10^{-5}$	$\pm 6.41 \times 10^{-5}$
0.93	2.803×10^{-3}	6.126×10^{-3}	6.819×10^{-3}	6.996×10^{-3}
	$\pm 6.17 \times 10^{-5}$	$\pm 1.38 \times 10^{-4}$	$\pm 1.80 \times 10^{-4}$	$\pm 2.08 \times 10^{-4}$
0.95	3.815×10^{-3}	8.589×10^{-3}	9.951×10^{-3}	1.004×10^{-2}
	$\pm 6.80 \times 10^{-5}$	$\pm 1.58 \times 10^{-4}$	$\pm 1.94 \times 10^{-4}$	$\pm 2.10 \times 10^{-4}$
0.97	5.549×10^{-3}	1.234×10^{-2}	1.564×10^{-2}	1.555×10^{-2}
	$\pm 1.04 \times 10^{-4}$	$\pm 2.19 \times 10^{-4}$	$\pm 3.79 \times 10^{-4}$	$\pm 3.88 \times 10^{-4}$
0.98	6.881×10^{-3}	1.536×10^{-2}	2.036×10^{-2}	2.012×10^{-2}
	$\pm 1.64 \times 10^{-4}$	$\pm 3.16 \times 10^{-4}$	$\pm 4.65 \times 10^{-4}$	$\pm 4.71 \times 10^{-4}$

Table 22: A comparison of the efficiency of QL, HOL, HOL_r, and HOL_s as a function of the traffic intensity ρ, for sinusoidal arrival rates with $\bar{\lambda}$ and μ corresponding to a mean service time of 30 minutes. Point and 95% confidence interval estimates of the ASE's are shown.

Efficiency of QL_m, QL_h, and HOL in the $M_t/M/s+M$ Model

s	QL_m	QL_h	HOL
100	3.059×10^{-3}	5.556×10^{-3}	1.623×10^{-2}
	$\pm 1.95 \times 10^{-4}$	$\pm 4.23 \times 10^{-4}$	$\pm 9.52 \times 10^{-4}$
300	9.911×10^{-4}	1.630×10^{-3}	1.114×10^{-2}
	$\pm 7.07 \times 10^{-5}$	$\pm 1.43 \times 10^{-4}$	$\pm 4.33 \times 10^{-4}$
500	5.474×10^{-4}	9.653×10^{-4}	1.033×10^{-2}
	$\pm 4.42 \times 10^{-5}$	$\pm 6.37 \times 10^{-5}$	$\pm 2.34 \times 10^{-4}$
700	4.076×10^{-4}	6.780×10^{-4}	9.866×10^{-3}
	$\pm 2.08 \times 10^{-5}$	$\pm 3.09 \times 10^{-5}$	$\pm 2.26 \times 10^{-4}$
1000	2.853×10^{-4}	4.907×10^{-4}	9.806×10^{-3}
	$\pm 2.48 \times 10^{-5}$	$\pm 1.90 \times 10^{-5}$	$\pm 1.75 \times 10^{-4}$

Table 23: A comparison of the efficiency of QL_m, QL_h, and HOL as a function of the number of servers s, for sinusoidal arrival rates with $\bar{\lambda}$ and μ corresponding to a mean service time of 6 hours. Point and 95% confidence interval estimates of the ASE's are shown.

Efficiency of QL_m, QL_h, and HOL in the $M_t/M/s+H_2$ Model

s	QL_m	QL_h	HOL
100	3.166×10^{-3}	3.710×10^{-3}	7.951×10^{-3}
	$\pm 1.52 \times 10^{-4}$	$\pm 1.77 \times 10^{-4}$	$\pm 5.38 \times 10^{-4}$
300	1.488×10^{-3}	1.148×10^{-3}	4.768×10^{-3}
	$\pm 5.61 \times 10^{-5}$	$\pm 7.93 \times 10^{-5}$	$\pm 2.16 \times 10^{-4}$
500	1.192×10^{-3}	7.139×10^{-4}	4.227×10^{-3}
	$\pm 5.26 \times 10^{-5}$	$\pm 4.74 \times 10^{-5}$	$\pm 1.85 \times 10^{-4}$
700	1.067×10^{-3}	5.180×10^{-4}	3.960×10^{-3}
	$\pm 4.18 \times 10^{-5}$	$\pm 2.92 \times 10^{-5}$	$\pm 1.31 \times 10^{-4}$
1000	9.590×10^{-4}	3.363×10^{-4}	3.827×10^{-3}
	$\pm 2.24 \times 10^{-5}$	$\pm 1.86 \times 10^{-5}$	$\pm 5.75 \times 10^{-5}$

Table 24: A comparison of the efficiency of QL_m, QL_h, and HOL as a function of the number of servers s, for sinusoidal arrival rates with $\bar{\lambda}$ and μ corresponding to a mean service time of 6 hours. Point and 95% confidence interval estimates of the ASE's are shown.

Efficiency of QL_m, QL_h, and HOL in the $M_t/M/s + E_{10}$ Model

s	QL_m	QL_h	HOL
100	9.542×10^{-3}	6.481×10^{-3}	4.531×10^{-2}
	$\pm 4.26 \times 10^{-4}$	$\pm 2.80 \times 10^{-4}$	$\pm 1.79 \times 10^{-3}$
300	7.711×10^{-3}	2.551×10^{-3}	3.744×10^{-2}
	$\pm 2.94 \times 10^{-4}$	$\pm 9.64 \times 10^{-5}$	$\pm 9.55 \times 10^{-4}$
500	7.083×10^{-3}	1.666×10^{-3}	3.652×10^{-2}
	$\pm 2.63 \times 10^{-4}$	$\pm 8.36 \times 10^{-5}$	$\pm 8.56 \times 10^{-4}$
700	6.875×10^{-3}	1.360×10^{-3}	3.622×10^{-2}
	$\pm 1.73 \times 10^{-4}$	$\pm 6.38 \times 10^{-5}$	$\pm 5.22 \times 10^{-4}$
1000	6.858×10^{-3}	1.070×10^{-3}	3.582×10^{-2}
	$\pm 1.17 \times 10^{-4}$	$\pm 4.27 \times 10^{-5}$	$\pm 6.03 \times 10^{-4}$

Table 25: A comparison of the efficiency of QL_m, QL_h, and HOL as a function of the number of servers s, for sinusoidal arrival rates with $\bar{\lambda}$ and μ corresponding to a mean service time of 6 hours. Point and 95% confidence interval estimates of the ASE's are shown.

Efficiency of QL_m, QL_h, and HOL in the $M_t/D/s + M$ Model

s	QL_m	QL_h	HOL
100	4.160×10^{-3}	6.038×10^{-3}	1.740×10^{-2}
	$\pm 7.26 \times 10^{-4}$	$\pm 7.40 \times 10^{-4}$	$\pm 9.66 \times 10^{-4}$
300	2.909×10^{-3}	3.552×10^{-3}	1.378×10^{-2}
	$\pm 3.78 \times 10^{-4}$	$\pm 3.97 \times 10^{-4}$	$\pm 5.69 \times 10^{-4}$
500	2.729×10^{-3}	3.181×10^{-3}	1.319×10^{-2}
	$\pm 6.33 \times 10^{-4}$	$\pm 6.53 \times 10^{-4}$	$\pm 8.80 \times 10^{-4}$
700	2.730×10^{-3}	2.971×10^{-3}	1.277×10^{-2}
	$\pm 2.97 \times 10^{-4}$	$\pm 2.91 \times 10^{-4}$	$\pm 4.14 \times 10^{-4}$
1000	2.963×10^{-3}	3.165×10^{-3}	1.286×10^{-2}
	$\pm 4.25 \times 10^{-4}$	$\pm 4.57 \times 10^{-4}$	$\pm 5.83 \times 10^{-4}$

Table 26: A comparison of the efficiency of QL_m, QL_h, and HOL as a function of the number of servers s, for sinusoidal arrival rates with $\bar{\lambda}$ and μ corresponding to a mean service time of 6 hours. Point and 95% confidence interval estimates of the ASE's are shown.

Efficiency of QL_m, QL_h, and HOL in the $M_t/H_2/s+M$ Model			
s	QL_m	QL_h	HOL
100	3.927×10^{-3}	5.981×10^{-3}	1.618×10^{-2}
	$\pm 5.39 \times 10^{-4}$	$\pm 6.63 \times 10^{-4}$	$\pm 1.22 \times 10^{-3}$
300	1.214×10^{-3}	1.885×10^{-3}	1.086×10^{-2}
	$\pm 9.46 \times 10^{-5}$	$\pm 1.14 \times 10^{-4}$	$\pm 5.24 \times 10^{-4}$
500	7.404×10^{-4}	1.137×10^{-3}	1.018×10^{-2}
	$\pm 5.91 \times 10^{-5}$	$\pm 8.52 \times 10^{-5}$	$\pm 6.05 \times 10^{-4}$
700	5.542×10^{-4}	7.844×10^{-4}	9.763×10^{-3}
	$\pm 3.36 \times 10^{-5}$	$\pm 3.50 \times 10^{-5}$	$\pm 2.65 \times 10^{-4}$
1000	3.760×10^{-4}	5.655×10^{-4}	9.189×10^{-3}
	$\pm 2.56 \times 10^{-5}$	$\pm 2.82 \times 10^{-5}$	$\pm 2.42 \times 10^{-4}$

Table 27: A comparison of the efficiency of QL_m, QL_h, and HOL as a function of the number of servers s, for sinusoidal arrival rates with $\bar{\lambda}$ and μ corresponding to a mean service time of 6 hours. Point and 95% confidence interval estimates of the ASE's are shown.

Efficiency of QL_m, QL_h, and HOL in the $M_t/D/s+H_2$ Model			
s	QL_m	QL_h	HOL
100	4.675×10^{-3}	4.959×10^{-3}	9.991×10^{-3}
	$\pm 3.20 \times 10^{-4}$	$\pm 3.60 \times 10^{-4}$	$\pm 5.07 \times 10^{-4}$
300	3.732×10^{-3}	3.158×10^{-3}	7.574×10^{-3}
	$\pm 1.50 \times 10^{-4}$	$\pm 1.35 \times 10^{-4}$	$\pm 2.48 \times 10^{-4}$
500	3.454×10^{-3}	2.723×10^{-3}	7.044×10^{-3}
	$\pm 1.84 \times 10^{-4}$	$\pm 1.59 \times 10^{-4}$	$\pm 3.08 \times 10^{-4}$
700	3.309×10^{-3}	2.552×10^{-3}	6.783×10^{-3}
	$\pm 1.19 \times 10^{-4}$	$\pm 1.19 \times 10^{-4}$	$\pm 1.96 \times 10^{-4}$
1000	3.269×10^{-3}	2.433×10^{-3}	6.585×10^{-3}
	$\pm 7.00 \times 10^{-5}$	$\pm 7.07 \times 10^{-5}$	$\pm 1.04 \times 10^{-4}$

Table 28: A comparison of the efficiency of QL_m, QL_h, and HOL as a function of the number of servers s, for sinusoidal arrival rates with $\bar{\lambda}$ and μ corresponding to a mean service time of 6 hours. Point and 95% confidence interval estimates of the ASE's are shown.

Efficiency of QL_m, QL_h, and HOL in the $M_t/H_2/s+H_2$ Model			
s	QL_m	QL_h	HOL
100	3.307×10^{-3}	3.972×10^{-3}	7.816×10^{-3}
	$\pm 2.10 \times 10^{-4}$	$\pm 3.58 \times 10^{-4}$	$\pm 6.48 \times 10^{-4}$
300	1.642×10^{-3}	1.285×10^{-3}	4.636×10^{-3}
	$\pm 1.38 \times 10^{-4}$	$\pm 9.20 \times 10^{-5}$	$\pm 2.87 \times 10^{-4}$
500	1.282×10^{-3}	7.739×10^{-4}	3.985×10^{-3}
	$\pm 9.09 \times 10^{-5}$	$\pm 3.40 \times 10^{-5}$	$\pm 1.79 \times 10^{-4}$
700	1.155×10^{-3}	5.510×10^{-4}	3.862×10^{-3}
	$\pm 8.24 \times 10^{-5}$	$\pm 3.34 \times 10^{-5}$	$\pm 1.50 \times 10^{-4}$
1000	1.099×10^{-3}	4.310×10^{-4}	3.557×10^{-3}
	$\pm 5.33 \times 10^{-5}$	$\pm 1.80 \times 10^{-5}$	$\pm 8.50 \times 10^{-5}$

Table 29: A comparison of the efficiency of QL_m, QL_h, and HOL as a function of the number of servers s, for sinusoidal arrival rates with $\bar{\lambda}$ and μ corresponding to a mean service time of 6 hours. Point and 95% confidence interval estimates of the ASE's are shown.

Efficiency of QL_m, QL_h, and HOL in the $M_t/D/s+E_{10}$ Model			
s	QL_m	QL_h	HOL
100	2.341×10^{-2}	2.182×10^{-2}	6.829×10^{-2}
	$\pm 3.82 \times 10^{-3}$	$\pm 4.15 \times 10^{-3}$	$\pm 7.04 \times 10^{-3}$
300	2.355×10^{-2}	2.101×10^{-2}	6.645×10^{-2}
	$\pm 1.60 \times 10^{-3}$	$\pm 1.78 \times 10^{-3}$	$\pm 2.19 \times 10^{-3}$
500	2.433×10^{-2}	2.119×10^{-2}	6.674×10^{-2}
	$\pm 1.97 \times 10^{-3}$	$\pm 2.16 \times 10^{-3}$	$\pm 3.55 \times 10^{-3}$
700	2.374×10^{-2}	2.033×10^{-2}	6.585×10^{-2}
	$\pm 1.10 \times 10^{-3}$	$\pm 1.29 \times 10^{-3}$	$\pm 1.86 \times 10^{-3}$
1000	2.372×10^{-2}	1.997×10^{-2}	6.492×10^{-2}
	$\pm 8.48 \times 10^{-4}$	$\pm 9.19 \times 10^{-4}$	$\pm 1.33 \times 10^{-3}$

Table 30: A comparison of the efficiency of QL_m, QL_h, and HOL as a function of the number of servers s, for sinusoidal arrival rates with $\bar{\lambda}$ and μ corresponding to a mean service time of 6 hours. Point and 95% confidence interval estimates of the ASE's are shown.

Efficiency of QL_m, QL_h, and HOL in the $M_t/H_2/s + E_{10}$ Model

s	QL_m	QL_h	HOL
100	9.970×10^{-3} $\pm 6.08 \times 10^{-4}$	7.130×10^{-3} $\pm 4.09 \times 10^{-4}$	4.190×10^{-2} $\pm 1.82 \times 10^{-3}$
300	7.599×10^{-3} $\pm 2.80 \times 10^{-4}$	3.025×10^{-3} $\pm 2.18 \times 10^{-4}$	3.746×10^{-2} $\pm 9.82 \times 10^{-4}$
500	7.097×10^{-3} $\pm 2.57 \times 10^{-4}$	1.983×10^{-3} $\pm 8.23 \times 10^{-5}$	3.589×10^{-2} $\pm 3.96 \times 10^{-4}$
700	6.373×10^{-3} $\pm 1.70 \times 10^{-4}$	1.741×10^{-3} $\pm 1.01 \times 10^{-4}$	3.554×10^{-2} $\pm 3.13 \times 10^{-4}$
1000	6.548×10^{-3} $\pm 1.72 \times 10^{-4}$	1.016×10^{-3} $\pm 7.86 \times 10^{-5}$	3.469×10^{-2} $\pm 5.58 \times 10^{-4}$

Table 31: A comparison of the efficiency of QL_m, QL_h, and HOL as a function of the number of servers s, for sinusoidal arrival rates with $\bar{\lambda}$ and μ corresponding to a mean service time of 6 hours. Point and 95% confidence interval estimates of the ASE's are shown.

$M_t/M/100$, $\alpha = 0.1$, $E[S] = 5$ *min*

	$HOL_r(x)$								
ρ	$x = 0.1$	$x = 0.05$	$x = 0.02$	HOL_r	$x =$ -0.02	$x =$ -0.05	$x =$ -0.1	QL	HOL
0.9	9.475×10^{-3} $\pm 8.17 \times 10^{-4}$	7.181×10^{-3} $\pm 5.26 \times 10^{-4}$	6.598×10^{-3} $\pm 4.74 \times 10^{-4}$	6.479×10^{-3} $\pm 4.81 \times 10^{-4}$	6.548×10^{-3} $\pm 5.16 \times 10^{-4}$	6.957×10^{-3} $\pm 6.17 \times 10^{-4}$	8.309×10^{-3} $\pm 8.88 \times 10^{-4}$	3.372×10^{-3} $\pm 2.44 \times 10^{-4}$	6.739×10^{-3} $\pm 5.02 \times 10^{-4}$
0.93	4.109×10^{-2} $\pm 1.54 \times 10^{-3}$	2.415×10^{-2} $\pm 7.49 \times 10^{-4}$	2.007×10^{-2} $\pm 5.68 \times 10^{-4}$	1.941×10^{-2} $\pm 5.75 \times 10^{-4}$	2.019×10^{-2} $\pm 6.74 \times 10^{-4}$	2.370×10^{-2} $\pm 9.49 \times 10^{-4}$	3.466×10^{-2} $\pm 1.64 \times 10^{-4}$	9.789×10^{-3} $\pm 2.79 \times 10^{-4}$	2.147×10^{-2} $\pm 7.55 \times 10^{-4}$
0.95	0.1021 $\pm 3.28 \times 10^{-3}$	5.067×10^{-2} $\pm 1.36 \times 10^{-3}$	3.823×10^{-2} $\pm 9.63 \times 10^{-4}$	3.620×10^{-2} $\pm 9.82 \times 10^{-4}$	3.853×10^{-2} $\pm 1.23 \times 10^{-3}$	4.911×10^{-2} $\pm 1.94 \times 10^{-3}$	8.229×10^{-2} $\pm 3.81 \times 10^{-3}$	1.821×10^{-2} $\pm 4.46 \times 10^{-4}$	4.667×10^{-2} $\pm 1.88 \times 10^{-3}$
0.97	0.2212 $\pm 4.13 \times 10^{-3}$	9.449×10^{-2} $\pm 1.82 \times 10^{-3}$	6.337×10^{-2} $\pm 1.39 \times 10^{-3}$	5.788×10^{-2} $\pm 1.53 \times 10^{-3}$	6.300×10^{-2} $\pm 2.097 \times 10^{-3}$	8.799×10^{-2} $\pm 3.6 \times 10^{-3}$	0.1674838 $\pm 7.27 \times 10^{-3}$	2.919×10^{-2} $\pm 1.03 \times 10^{-3}$	9.586×10^{-2} $\pm 3.06 \times 10^{-3}$
0.98	0.3087 $\pm 6.43 \times 10^{-3}$	0.1245 $\pm 2.71 \times 10^{-3}$	7.894×10^{-2} $\pm 1.90 \times 10^{-3}$	7.068×10^{-2} $\pm 1.85 \times 10^{-3}$	7.777×10^{-2} $\pm 2.18 \times 10^{-3}$	0.1135 $\pm 3.25 \times 10^{-3}$	0.2277 $\pm 6.264 \times 10^{-3}$	3.523×10^{-2} $\pm 9.44 \times 10^{-4}$	0.1354 $\pm 3.81 \times 10^{-3}$
Sample size Est. interval	385 (20 min.)	1537 (77 min.)	9604 (480 min.)		9604 (480 min.)	1537 (77 min.)	385 (20 min.)		

Table 32: Performance of $\text{HOL}_r(x)$ delay estimators, as a function of the traffic intensity, ρ, and alternative x, in the $M_t/M/100$ queueing model with $\alpha = 0.1$ and $E[S] = 5$ minutes. Sample sizes needed and length of estimation intervals required are also included.

$M_t/M/100$, $\alpha = 0.5$, $E[S] = 5$ *min*

	$HOL_r(x)$								
ρ	$x = 0.1$	$x = 0.05$	$x = 0.02$	HOL$_r$	$x =$ -0.02	$x =$ -0.05	$x =$ -0.1	QL	HOL
0.9	4.40 $\pm 5.3\times 10^{-2}$	1.24 $\pm 2.53\times 10^{-2}$	0.449 $\pm 1.21\times 10^{-2}$	0.302 $\pm 6.4\times 10^{-3}$	0.417 $\pm 9.3\times 10^{-3}$	1.02 $\pm 2.1\times 10^{-2}$	2.96 $\pm 4.1\times 10^{-2}$	0.148 $\pm 6.8\times 10^{-3}$	16.92 $\pm 1.4\times 10^{-1}$
0.93	6.01 $\pm 5.0\times 10^{-2}$	1.63 $\pm 2.9\times 10^{-2}$	0.548 $\pm 1.5\times 10^{-2}$	0.351 $\pm 8.8\times 10^{-3}$	0.520 $\pm 1.5\times 10^{-2}$	1.37 $\pm 3.4\times 10^{-2}$	4.09 $\pm 7.2\times 10^{-2}$	0.177 $\pm 6.0\times 10^{-3}$	28.0 ± 0.27
0.95	7.29 $\pm 9.3\times 10^{-2}$	1.96 $\pm 3.7\times 10^{-2}$	0.645 $\pm 1.7\times 10^{-2}$	0.410 $\pm 1.8\times 10^{-2}$	0.620 $\pm 2.8\times 10^{-2}$	1.66 $\pm 4.5\times 10^{-2}$	4.98 $\pm 7.1\times 10^{-2}$	0.202 $\pm 7.4\times 10^{-3}$	38.06 ± 0.32
0.97	8.48 ± 0.12	2.21 $\pm 5.5\times 10^{-2}$	0.688 $\pm 2.4\times 10^{-2}$	0.431 $\pm 1.4\times 10^{-2}$	0.702 $\pm 2.7\times 10^{-2}$	1.97 $\pm 5.7\times 10^{-2}$	5.96 ± 0.11	0.216 $\pm 6.6\times 10^{-3}$	49.8 ± 0.43
0.98 $\pm 8.2\times 10^{-2}$	9.21 $\pm 3.5\times 10^{-2}$	2.40 $\pm 2.3\times 10^{-2}$	0.741 $\pm 2.3\times 10^{-2}$	0.454 $\pm 3.0\times 10^{-2}$	0.737 $\pm 4.4\times 10^{-2}$	2.09 $\pm 7.4\times 10^{-2}$	6.39 $\pm 6.9\times 10^{-3}$	0.226 0.40	56.3
Sample size Est. interval	385 (20 min.)	1537 (77 min.)	9604 (480 min.)		9604 (480 min.)	1537 (77 min.)	385 (20 min.)		

Table 33: Performance of HOL$_r(x)$ delay estimators, as a function of the traffic intensity, ρ, and alternative x, in the $M_t/M/100$ queueing model with $\alpha = 0.5$ and $E[S] = 5$ minutes. Sample sizes needed and length of estimation intervals required are also included.

$M_t/H_2/100$, $\alpha = 0.1$, $E[S] = 5$ *min*

	$HOL_r(x)$								
ρ	$x = 0.1$	$x = 0.05$	$x = 0.02$	HOL$_r$	$x =$ -0.02	$x =$ -0.05	$x =$ -0.1	QL	HOL
0.9	0.02387831 0.00193371	0.01769823 0.001375825	0.01681182 0.001438474	0.01717618 0.001615818	0.01820248 0.001887946	0.02081818 0.002450358	0.02752088 0.0037207	0.01097116 0.001002839	0.01768422 0.001662133
0.93	0.07123022 0.005205414	0.04444417 0.002910188	0.03872027 0.002528451	0.03841576 0.002623301	0.04054891 0.002953477	0.04771991 0.003813547	0.06834477 0.006017725	0.02635588 0.00169279	0.04314007 0.003260498
0.95	0.15442815 0.007988869	0.08696279 0.003969364	0.07192414 0.003560726	0.07053472 0.003909899	0.07514229 0.004762332	0.09182688 0.00695044	0.14098948 0.012584191	0.05056835 0.003338725	0.08874154 0.006426826
0.97	0.29321013 0.014053147	0.15109947 0.006076998	0.11744914 0.004947495	0.11254818 0.005496626	0.11982738 0.007025441	0.15060582 0.010892683	0.24533181 0.020616258	0.08128917 0.004183756	0.16170994 0.011787138
0.98	0.4362757 0.017287792	0.21070649 0.00802167	0.15823673 0.005632002	0.15140102 0.005381568	0.16411467 0.006140708	0.21505534 0.008664614	0.36963231 0.015346337	0.1125588 0.004236825	0.24656124 0.01206609
Sample size Est. interval	1537 (76 min.)	6147 (307 min.)	38416 (1920 min.)		38416 (1920 min.)	6147 (307 min.)	1537 (76 min.)		

Table 34: Performance of HOL$_r(x)$ delay estimators, as a function of the traffic intensity, ρ, and alternative x, in the $M_t/H_2/100$ queueing model with $\alpha = 0.1$ and $E[S] = 5$ minutes. Sample sizes needed and length of estimation intervals required are also included.

$M_t/H_2/100$, $\alpha = 0.5$, $E[S]\ = 5\ min$

	$HOL_r(x)$								
ρ	$x = 0.1$	$x = 0.05$	$x = 0.02$	HOL_r	$x =$ -0.02	$x =$ -0.05	$x =$ -0.1	QL	HOL
0.9	4.82938576	1.62351301	0.83549186	0.69600326	0.82466213	1.45501205	3.46253145	0.54968671	17.92757786
1/2 width conf. int.	0.05528957	0.034127039	0.025332142	0.024316198	0.031148139	0.053052519	0.105926474	0.017114564	0.416601257
0.93	6.49207791	2.10909787	1.03921508	0.8559942	1.04109108	1.91944062	4.69754385	0.6660811	29.04497638
1/2 width conf. int.	0.073417678	0.044674609	0.036163933	0.036411958	0.042572653	0.060697353	0.10494574	0.025490489	0.444163211
0.95	7.76875124	2.45240809	1.14814297	0.91935046	1.13580851	2.18670745	5.52709242	0.72815548	38.62497431
1/2 width conf. int.	0.207046777	0.098246756	0.045493678	0.027428619	0.047165922	0.098950036	0.195825838	0.026756631	0.814421988
0.97	9.1684113	2.84852199	1.29216257	1.01428381	1.2643532	2.50058171	6.44513529	0.78872996	50.30361131
1/2 width conf. int.	0.192079696	0.109273562	0.063127366	0.046738323	0.061153914	0.116201171	0.231838052	0.035678163	0.947957048
0.98	9.70672551	2.912211867	1.290429389	1.034932733	1.353123856	2.765918733	7.1667349	0.818581556	57.87150467
1/2 width conf. int.	0.155481569	0.66280096	0.295143427	0.237006597	0.310534097	0.633130784	1.632297322	0.187588026	13.12175948
Sample size	1537	6147	38416		38416	6147	1537		
Est. interval	(76 min.)	(307 min.)	(1920 min.)		(1920 min.)	(307 min.)	(76 min.)		

Table 35: Performance of $\text{HOL}_r(x)$ delay estimators, as a function of the traffic intensity, ρ, and alternative x, in the $M_t/H_2/100$ queueing model with $\alpha = 0.5$ and $E[S] = 5$ minutes. Sample sizes needed and length of estimation intervals required are also included.

$M_t/D/100$, $\alpha = 0.1$, $E[S]\ = 5\ min$

	$HOL_r(x)$								
ρ	$x = 0.1$	$x = 0.05$	$x = 0.02$	HOL_r	$x =$ -0.02	$x =$ -0.05	$x =$ -0.1	QL	HOL
0.9	0.00434374	0.00329361	0.00297954	0.00287784	0.00285117	0.00293378	0.00334088	0.00100706	0.00301744
1/2 width conf. int.	0.000232398	0.000130592	0.000109445	0.000108738	0.000117295	0.000145377	0.000225701	2.96554×10^{-5}	0.000114647
0.93	0.02321957	0.01232221	0.00949884	0.0088798	0.00913909	0.01096134	0.01713719	0.0012039	0.00979293
1/2 width conf. int.	0.001372613	0.000483134	0.000313483	0.000328706	0.000438206	0.000750349	0.001581422	3.74×10^{-5}	0.000498869
0.95	0.07018926	0.02938626	0.01896676	0.01680122	0.01795916	0.02511899	0.04892439	0.00129737	0.02323254
1/2 width conf. int.	0.000838908	0.000379223	0.000266873	0.000256304	0.000333164	0.000590254	0.001250683	3.7079×10^{-5}	0.000443064
0.97	0.17752475	0.06274624	0.03408225	0.02863705	0.03268889	0.05425917	0.12411535	0.00127323	0.06024986
1/2 width conf. int.	0.006029715	0.001888735	0.000871529	0.000811559	0.001245078	0.00247533	0.005721494	5.5966×10^{-5}	0.003229276
0.98	0.25040185	0.08526632	0.04316647	0.0344715	0.03924311	0.06837323	0.16502711	0.00128441	0.08662659
1/2 width conf. int.	0.003422875	0.001114701	0.000478799	0.000426636	0.000570489	0.00091089	0.00187367	6.58892×10^{-5}	0.001562013
Sample size	385	1537	9604		9604	1537	385		
Est. interval	(20 min.)	(77 min.)	(480 min.)		(480 min.)	(77 min.)	(20 min.)		

Table 36: Performance of $\text{HOL}_r(x)$ delay estimators, as a function of the traffic intensity, ρ, and alternative x, in the $M_t/D/100$ queueing model with $\alpha = 0.1$ and $E[S] = 5$ minutes. Sample sizes needed and length of estimation intervals required are also included.

$M_t/D/100$, $\alpha = 0.5$, $E[S] = 5$ *min*									
	$HOL_r(x)$								
ρ	$x = 0.1$	$x = 0.05$	$x = 0.02$	HOL_r	$x =$ -0.02	$x =$ -0.05	$x =$ -0.1	QL	HOL
0.9	4.22570671	1.0823633	0.29770931	0.14892846	0.26031388	0.85179977	2.76642575	0.0013661	16.54569602
	0.041226478	0.016272198	0.005245399	0.003305968	0.008247814	0.015643502	0.026150156	4.78367E-05	0.08637948
0.93	5.78589497	1.43540134	0.36950715	0.18369967	0.36257659	1.2256914	3.96555789	0.00131948	27.92663528
	0.091515404	0.041767531	0.013192279	0.00691783	0.026013066	0.055831955	0.107541493	4.62817E-05	0.402777602
0.95	7.03332018	1.74621982	0.43735389	0.19803505	0.39881881	1.41787246	4.68716641	0.00138821	37.20827391
	0.021453459	0.009310012	0.007546074	0.007854712	0.008607295	0.010122413	0.014340562	6.04662E-05	0.115009123
0.97	8.40047517	2.09017293	0.51445635	0.2152629	0.43823876	1.62451735	5.46585413	0.00138931	48.34117758
	0.070380898	0.031558519	0.013209368	0.003774982	0.009212359	0.023241922	0.045791655	8.61113E-05	0.21721625
0.98	9.01628391	2.21980016	0.53108293	0.21724639	0.46773577	1.7640093	5.93891838	0.00136874	54.83274893
	0.071760275	0.033824992	0.01439316	0.005462084	0.012351064	0.028721951	0.055376387	7.9147E-05	0.158699284
Sample size	385	1537	9604		9604	1537	385		
Est. interval	(20 min.)	(77 min.)	(480 min.)		(480 min.)	(77 min.)	(20 min.)		

Table 37: Performance of $\text{HOL}_r(x)$ delay estimators, as a function of the traffic intensity, ρ, and alternative x, in the $M_t/D/100$ queueing model with $\alpha = 0.5$ and $E[S] = 5$ minutes. Sample sizes needed and length of estimation intervals required are also included.

$M_t/M/100$, $\alpha = 0.1$, $E[S]$ = *30 min*

	$HOL_r(x)$								
ρ	x = 0.1	x = 0.05	x = 0.02	HOL_r	x = -0.02	x = -0.05	x = -0.1	QL	HOL
0.9	0.0054772	0.00456605	0.00433673	0.00429175	0.00432173	0.00448906	0.00503571	0.00225791	0.00461371
1/2 width conf. int.	0.000111799	9.03E-05	8.67455E-05	8.76143E-05	9.08199E-05	9.94898E-05	0.000122413	5.11946E-05	9.78569E-05
0.93	0.01023003	0.00794537	0.00738668	0.00729019	0.00738545	0.00784111	0.0092848	0.00376822	0.00804103
1/2 width conf. int.	0.000286275	0.000213791	0.000205303	0.000211852	0.000227042	0.000263957	0.0003559	0.000104091	0.000257155
0.95	0.01543503	0.01126469	0.01024003	0.01005901	0.01022693	0.01104788	0.01366132	0.00507745	0.01165303
1/2 width conf. int.	0.000179156	0.000137672	0.000137791	0.000146471	0.000161424	0.000194651	0.000274113	7.24005E-05	0.000198268
0.97	0.02418008	0.01636038	0.01444975	0.01412101	0.01444895	0.01601191	0.02096003	0.00716086	0.01750448
1/2 width conf. int.	0.000288114	0.000204498	0.000193653	0.000199194	0.000215457	0.000260922	0.000387624	9.84017E-05	0.000244767
0.98	0.03411682	0.02160732	0.01853606	0.01799541	0.01850195	0.02096969	0.02881951	0.00913619	0.02390237
1/2 width conf. int.	0.001323293	0.000690408	0.000574084	0.000587134	0.000662906	0.000877217	0.001451698	0.000302351	0.001019484
Sample size	385	1537	9604		9604	1537	385		
Est. interval	(20 min.)	(77 min.)	(480 min.)		(480 min.)	(77 min.)	(20 min.)		

Table 38: Performance of $\text{HOL}_r(x)$ delay estimators, as a function of the traffic intensity, ρ, and alternative x, in the $M_t/M/100$ queueing model with $\alpha = 0.1$ and $E[S] = 30$ minutes. Sample sizes needed and length of estimation intervals required are also included.

$M_t/H_2/100$, $\alpha = 0.1$, $E[S]$ = *30 min*

	$HOL_r(x)$								
ρ	x = 0.1	x = 0.05	x = 0.02	HOL_r	x = -0.02	x = -0.05	x = -0.1	QL	HOL
0.9	0.00908255	0.00789063	0.00785373	0.00805817	0.00842098	0.00922217	0.01111496	0.00491717	0.00864924
	0.000331025	0.000330445	0.000363243	0.000396554	0.000437969	0.000513361	0.000667913	0.000238446	0.000451521
0.93	0.01532327	0.01245523	0.01215336	0.012432	0.01304293	0.01449914	0.01809981	0.00772666	0.01410048
	0.000348223	0.000303908	0.000326134	0.000355384	0.000393724	0.000465218	0.000614296	0.000254548	0.000408832
0.95	0.02517471	0.01915692	0.01829549	0.01865194	0.01965335	0.02220407	0.02873824	0.01198275	0.02226189
	0.000912695	0.00067449	0.000684651	0.000744573	0.000842085	0.001049847	0.001528719	0.000650097	0.000972634
0.97	0.04367883	0.03015133	0.02753872	0.02766327	0.02908263	0.03331972	0.04498074	0.01792297	0.03527407
	0.002905679	0.001751769	0.001601145	0.001678052	0.001875124	0.002366537	0.003620258	0.001253829	0.002282089
0.98	0.08337174	0.05145536	0.04480453	0.04461123	0.04736124	0.05628061	0.081614	0.03032998	0.06236974
	0.007783602	0.003819553	0.003110537	0.003176379	0.003628687	0.004973847	0.008702718	0.002326604	0.005831908
Sample size	1537	6147	38416		38416	6147	1537		
Est. interval	(76 min.)	(307 min.)	(1920 min.)		(1920 min.)	(307 min.)	(76 min.)		

Table 39: Performance of $\text{HOL}_r(x)$ delay estimators, as a function of the traffic intensity, ρ, and alternative x, in the $M_t/H_2/100$ queueing model with $\alpha = 0.1$ and $E[S] = 30$ minutes. Sample sizes needed and length of estimation intervals required are also included.

$M_t/D/100$, $\alpha = 0.1$, $E[S] = 30$ *min*

ρ	$x = 0.1$	$x = 0.05$	$x = 0.02$	$HOL_r(x)$ HOL$_r$	$x =$ -0.02	$x =$ -0.05	$x =$ -0.1	QL	HOL
0.9	0.00311196	0.00255554	0.00237535	0.00230769	0.00227663	0.00228992	0.00244394	0.00097235	0.00247478
	5.09448E-05	3.84498E-05	3.48937E-05	3.36869E-05	3.33585E-05	3.45003E-05	4.0955E-05	2.52873E-05	3.58366E-05
0.93	0.00593709	0.00441721	0.00397407	0.00383833	0.00381377	0.00395868	0.00459904	0.00123309	0.00417875
	0.000106993	7.18932E-05	6.34399E-05	6.25189E-05	6.52415E-05	7.58518E-05	0.000107663	2.42445E-05	7.81582E-05
0.95	0.009205	0.00623369	0.00541337	0.00519407	0.00520268	0.00558779	0.00704572	0.00131465	0.00600863
	4.63488E-05	3.93065E-05	3.97811E-05	4.10947E-05	4.295E-05	4.64267E-05	5.40293E-05	2.69204E-05	4.10268E-05
0.97	0.01509037	0.00921434	0.00764293	0.00726011	0.00733965	0.00821376	0.0113245	0.00135067	0.00929444
	9.69532E-05	7.51393E-05	6.78412E-05	6.51647E-05	6.46938E-05	6.9182E-05	9.13305E-05	2.64342E-05	3.81792E-05
0.98	0.01901157	0.01100473	0.00883741	0.00828971	0.00836603	0.00949945	0.01362158	0.00133603	0.01130859
	0.000101965	6.64308E-05	5.75469E-05	5.70556E-05	6.26205E-05	8.30851E-05	0.000141748	4.16968E-05	6.9318E-05
Sample size	385	1537	9604		9604	1537	385		
Est. interval	(20 min.)	(77 min.)	(480 min.)		(480 min.)	(77 min.)	(20 min.)		

Table 40: Performance of HOL$_r(x)$ delay estimators, as a function of the traffic intensity, ρ, and alternative x, in the $M_t/D/100$ queueing model with $\alpha = 0.1$ and $E[S] = 30$ minutes. Sample sizes needed and length of estimation intervals required are also included.

$M_t/M/100$, $\alpha = 0.5$, $E[S] = 6$ *hours*

ρ	$x = 0.1$	$x = 0.05$	$x = 0.02$	$HOL_r(x)$ HOL$_r$	$x =$ -0.02	$x =$ -0.05	$x =$ -0.1	QL	HOL
0.9	0.00518397	0.00447759	0.00429917	0.00426365	0.00428609	0.00441437	0.00483525	0.00223542	0.00899525
	4.70066E-05	4.83733E-05	5.21955E-05	5.56796E-05	5.97783E-05	6.67865E-05	8.02373E-05	2.38278E-05	0.000158959
0.93	0.00693836	0.00583694	0.00557005	0.00552602	0.00557498	0.00580016	0.00650725	0.00284982	0.01434114
	7.60619E-05	6.98512E-05	7.16117E-05	7.45035E-05	7.8551E-05	8.65175E-05	0.00010413	3.67747E-05	0.000243708
0.95	0.00907295	0.00734847	0.00693689	0.00687418	0.00695851	0.00732484	0.00845981	0.00352048	0.02190101
	0.000158923	0.000127762	0.000129945	0.000138512	0.000152075	0.000180514	0.000245455	4.76083E-05	0.00064789
0.97	0.01393218	0.01043122	0.00958005	0.00943713	0.00958918	0.01029833	0.01253267	0.00480099	0.03861194
	0.000485199	0.000272073	0.0002445	0.000258588	0.000295718	0.000390506	0.000635961	0.000115914	0.001422974
0.98	0.02044901	0.01422489	0.01267767	0.01238955	0.01261806	0.01380363	0.01762377	0.00634129	0.05967487
	0.000848817	0.000469432	0.000402572	0.000410311	0.000454951	0.000584634	0.000941804	0.000218413	0.002772512
Sample size	385	1537	9604		9604	1537	385		
Est. interval	(20 min.)	(77 min.)	(480 min.)		(480 min.)	(77 min.)	(20 min.)		

Table 41: Performance of HOL$_r(x)$ delay estimators, as a function of the traffic intensity, ρ, and alternative x, in the $M_t/M/100$ queueing model with $\alpha = 0.5$ and $E[S] = 6$ hours. Sample sizes needed and length of estimation intervals required are also included.

$M_t/H_2/100$, $\alpha = 0.5$, $E[S] = 6$ hours									
	$HOL_r(x)$								
ρ	$x = 0.1$	$x = 0.05$	$x = 0.02$	HOL$_r$	x = -0.02	x = -0.05	x = -0.1	QL	HOL
0.9	0.00620835	0.0054971	0.00539677	0.00544041	0.0055606	0.00586542	0.00664438	0.00318874	0.01295133
	0.000115183	0.000126597	0.000145492	0.000162068	0.000181337	0.000214562	0.000278959	0.00010977	0.000597349
0.93	0.00972352	0.00830506	0.00812898	0.00823998	0.0085092	0.00917016	0.01083109	0.00492565	0.0248648
	0.000254381	0.000240185	0.000258665	0.000280044	0.000307757	0.000359751	0.000469294	0.000160269	0.000909183
0.95	0.01581516	0.01271121	0.01232001	0.01255707	0.01313897	0.01457228	0.0181805	0.00767446	0.04345345
	0.000764233	0.00059119	0.000608856	0.000661228	0.000742507	0.000912978	0.001304908	0.000494673	0.00130275
0.97	0.03147157	0.02256362	0.02088109	0.02100101	0.02198223	0.0248562	0.03270458	0.01358017	0.07654237
	0.002249334	0.001472175	0.001370772	0.001422935	0.001559492	0.001906548	0.002804558	0.001034625	0.003179795
0.98	0.06633344	0.04177488	0.03676802	0.03673004	0.03898226	0.04609045	0.06607918	0.02465252	0.1134608
	0.009079108	0.004217227	0.00341951	0.00358856	0.004247994	0.006053269	0.010858913	0.002798193	0.004615704
Sample size	1537	6147	38416		38416	6147	1537		
Est. interval	(76 min.)	(307 min.)	(1920 min.)		(1920 min.)	(307 min.)	(76 min.)		

Table 42: Performance of HOL$_r(x)$ delay estimators, as a function of the traffic intensity, ρ, and alternative x, in the $M_t/H_2/100$ queueing model with $\alpha = 0.5$ and $E[S] = 6$ hours. Sample sizes needed and length of estimation intervals required are also included.

$M_t/D/100$, $\alpha = 0.5$, $E[S] = 6$ hours									
	$HOL_r(x)$								
ρ	$x = 0.1$	$x = 0.05$	$x = 0.02$	HOL$_r$	x = -0.02	x = -0.05	x = -0.1	QL	HOL
0.9	0.00728898	0.00606022	0.00565331	0.00549488	0.00541515	0.00542457	0.00572438	0.00308402	0.01057741
	1.35052E-05	1.34093E-05	1.45664E-05	1.56742E-05	1.69942E-05	1.92341E-05	2.34299E-05	1.34199E-05	4.82167E-05
0.93	0.00849714	0.00693075	0.00643117	0.00624837	0.00617036	0.00622482	0.00669295	0.00337701	0.01425161
	6.03262E-05	5.34889E-05	5.08546E-05	4.96357E-05	4.88037E-05	4.82428E-05	4.88753E-05	2.73109E-05	6.55912E-05
0.95	0.00940113	0.00751858	0.00692947	0.00672108	0.00664117	0.00673156	0.0073446	0.00351915	0.01765577
	4.68142E-05	3.79286E-05	3.4863E-05	3.38072E-05	3.35788E-05	3.47802E-05	4.05456E-05	2.31263E-05	0.000111196
0.97	0.01090106	0.00834069	0.00757214	0.00732143	0.00725297	0.0074483	0.00842842	0.00348867	0.02493805
	4.44251E-05	3.0013E-05	2.66165E-05	2.62961E-05	2.76932E-05	3.31078E-05	4.96838E-05	3.56664E-05	0.000291939
0.98	0.0126823	0.00920028	0.00819031	0.00788459	0.00783481	0.00817834	0.0096689	0.00336392	0.03400225
	0.000144308	8.27494E-05	7.38721E-05	7.67392E-05	8.58177E-05	0.000110134	0.000175379	4.86364E-05	0.000889717
Sample size	385	1537	9604		9604	1537	385		
Est. interval	(20 min.)	(77 min.)	(480 min.)		(480 min.)	(77 min.)	(20 min.)		

Table 43: Performance of HOL$_r(x)$ delay estimators, as a function of the traffic intensity, ρ, and alternative x, in the $M_t/D/100$ queueing model with $\alpha = 0.5$ and $E[S] = 6$ hours. Sample sizes needed and length of estimation intervals required are also included.

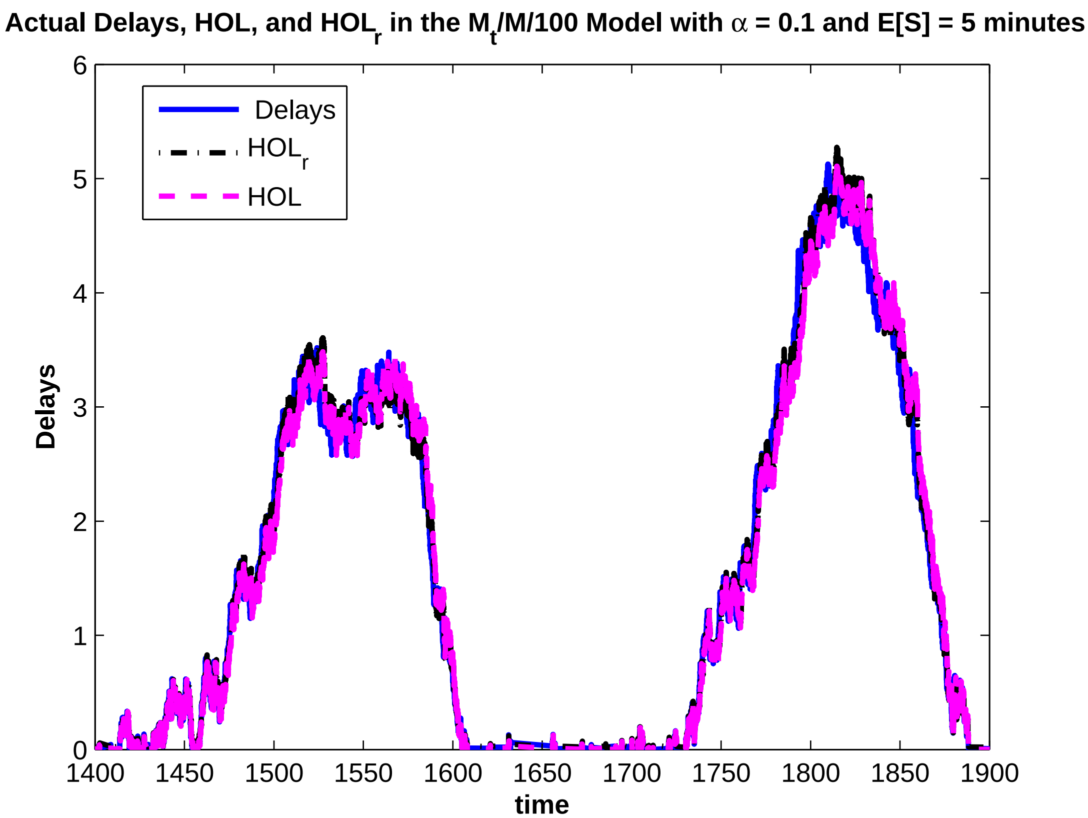

Figure 1: Sample paths of actual delays and corresponding delay estimates