Supplementary Material to the main paper:

A General Framework to Compare Announcement Accuracy: Static vs LES-based Announcement

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In this supplement to the main paper, we present additional simulation results in single-class and two-class systems, both with and without time-varying arrival rates. The results that we present here serve as robustness checks which further substantiate our theoretical results from the main paper. In §1, we describe steady-state simulations (where we discard the initial transient period); corresponding tables are in §3. In Figures 1-4, we consider smaller systems with 30 servers and illustrate that our results are reasonable in describing smaller systems too. In §2, we describe simulations based on short runs to consider the transient state of the system instead. We do so to investigate how fast the system converges to steady state. We also investigate how this convergence depends on the parameters of the system, such as the number of servers, the degree of impatience of customers, and the level of congestion in the system. The corresponding tables are in §4.

1. Description of the Results: Steady-State Simulations

Our simulation results, throughout this paper, are based on 10 independent replications of 2 million arrival events each. Our simulations are steady-state simulations. For this, we exclude from each simulation run the first 5,000 events so as to remove the effect of the initial transient period. We consider several distributions for the service and abandonment times: (i) Exponential (M); (ii) lognormal with mean a and variance b ($LN(a,b)$); (iii) Erlang (sum of two exponentials; E_2); (iv) deterministic (D); and (v) Hyper-exponential with balanced means (mixture of two exponentials; H_2). For our two-class models, we assume non-preemptive priority of the high-class over the low-class, and report results for the low-class. For models with time-varying arrivals, we focus on sinusoidal arrivals, i.e., we consider:

$$\lambda(u) = \bar{\lambda} + \bar{\lambda}\alpha \sin(\gamma u), \text{ for } 0 \leq u < \infty \ , \tag{1.1}$$

where $\bar{\lambda}$ is the average arrival rate and α is the relative amplitude.

For the WA prediction, we consider two alternatives: (i) we use the theoretical asymptotic expression for the correlation depending on the model as given in the main paper, and (ii) we consider a running-average simulation-based estimate for the correlation; the corresponding predictor is denoted "WA-run". We also consider a delay prediction which is equal to an exponentially smoothed average over previous LES delays, where we estimate the value of the smoothing factor in the simulation by using a gradient-descent method to minimize the errors between the smoothed averages and actual delays, in a training set consisting of 100 data points (after steady state is reached). We denote this predictor by "EXP".

1.1. More General Models

In Tables 1-4 we present point estimates of the ASE's and for the correlation between LES announcements and virtual delays of waiting customers in the single-class $M/G/100$ model for alternative service-time distributions. Tables 1-4 show that the WA and WA-run predictors are superior to both LES and EA throughout. We also observe that our theoretical results for the asymptotic form of the correlation are generally robust. They perform least well with deterministic service times (Table 3), yet remain useful also in that case when ρ is large enough. In Tables 5-10, we consider the $M/G/100+G$ model with alternative service and abandonment-time distributions. Our results are similar to the $M/G/s$ model in that WA and WA-run remain superior predictors, and the correlation estimates results are consistent with our theoretical expressions, which indicate that our asymptotic results are useful in describing more general systems as well. In Tables 11-18, we present simulation results for the two-class model with priorities, first without abandonment (Tables 11-12) then with abandonment (Tables 13-18). Once more, the results substantiate the superiority of the WA and WA-run predictions, and the usefulness of our asymptotic expressions for the correlations. In Tables 19 and 20, we consider the time-varying model. In Table 19, we fix $\rho_H = 5$ and $\rho_L = 0.2$. We also let $\alpha = 0.3$ and vary the value of the frequency γ. The results substantiate the superiority of the WA prediction. The correlation estimates are non-monotonic in the frequency γ, and an analysis of their values is left as a direction for future work. We see consistent results in Table 20 where we include abandonment to the time-varying model.

1.2. Small Number of Servers

In Figures 1-4, we consider smaller systems and plot the relative error (in percent, relative to the average virtual waiting time) of the LES and the EA announcements, as a function of the traffic intensity ρ in the system. For systems with two classes and priority (Figures 3 and 4) we fix ρ_H and vary ρ_L to vary the ratio $\rho_L/(1-\rho_H)$. We find that the asymptotic results on correlations of section 5 of the main paper describe smaller systems reasonably well too. Indeed, the relative performance of LES and EA is roughly the same as in larger systems in this case, and is related to the traffic intensity as predicted by the accompanying theory. For example, in Figure 1, our theory shows that the asymptotic form for the correlation is equal to ρ. Thus, based on Corollary 1 of the main paper, we expect that the LES predictor be less accurate than EA for $\rho \leq 0.5$ in large systems. From the figure, we see that this is roughly the case even when the system is small. For another example, in Figure 2, our asymptotic theory tells us that the asymptotic expression for the correlation is $1/\rho$. Thus, LES should be outperformed by EA when the system is heavily congested ($\rho > 2$). In the figure, we see that this is indeed roughly the case, even when the system is small.

2. Description of the Results: Transient Simulations

Our results, both numerical and theoretical, so far have focused on systems in steady state. In our empirical section, we approximated steady-state correlations by estimating correlations in a large enough sample of the data, to smooth out noise. In this section, we turn to studying the transient state of the system. Our objective is to form an understanding of how fast the system converges to steady state, depending on several parameters in the system. To this end, we consider $M/M/n+M$ systems where we vary the number of servers n, the arrival rate λ, and the abandonment rate θ. We vary the length of the simulation run and present point estimates of the ASE's for LES and EA, along with point estimates of the running correlation. In each case, we consider a single simulation run. In each case, we also present the relative percent error, compared to the theoretical value of the correlation (in all of the cases, equal to $1/\rho$). We make the following observations:

- Comparing Tables 21, 22, and 23, where we consider $n = 100, 500$ and 1000 respectively, while fixing all other parameters, we find that increasing the number of servers slows down convergence to steady state; see the error column in those tables.
- Comparing Tables 23, 24, and 25, where we consider $n = 1000$ and vary the traffic intensity ρ from 1.4 to 1.8, we find that convergence to steady state is slower in more congested systems; see the error column in those tables.
- Comparing Tables 23, 26, and 27, where we consider $n = 1000$, fix the traffic intensity $\rho = 1.4$ and vary the abandonment rate $\theta = 1$, 0.5 and 2 respectively (where higher θ corresponds to more impatient customers), we find that convergence to steady state is slower with more patient customers systems; see the error column in those tables. Having more patient customers leads to a more congested system, so that this is consistent with the second point above.

3. Steady State Simulations

3.1. Tables with Simulation Results for Single-Class Model

ρ	ASE(LES)	ASE(EA)	ASE(WA)	ASE(WA-run)	ASE(EXP)	r	$\mathbb{E}[W \mid W > 0]$
0.7	8.60×10^{-4} $\pm 5.8 \times 10^{-6}$	1.36×10^{-3} $\pm 8.1 \times 10^{-6}$	7.33×10^{-4} $\pm 6.1 \times 10^{-6}$	7.51×10^{-4} $\pm 7.6 \times 10^{-6}$	8.51×10^{-4} $\pm 6.5 \times 10^{-6}$	0.712 $\pm 1.3 \times 10^{-3}$	0.0370 $\pm 1.3 \times 10^{-4}$
0.75	9.81×10^{-4} $\pm 5.2 \times 10^{-6}$	1.65×10^{-3} $\pm 7.9 \times 10^{-6}$	8.40×10^{-4} $\pm 5.8 \times 10^{-6}$	8.42×10^{-4} $\pm 6.3 \times 10^{-6}$	9.73×10^{-4} $\pm 5.0 \times 10^{-6}$	0.726 $\pm 2.9 \times 10^{-4}$	0.0407 $\pm 1.0 \times 10^{-4}$
0.8	1.19×10^{-3} $\pm 2.0 \times 10^{-6}$	2.52×10^{-3} $\pm 9.0 \times 10^{-6}$	1.06×10^{-3} $\pm 2.3 \times 10^{-6}$	1.06×10^{-3} $\pm 2.7 \times 10^{-6}$	1.19×10^{-3} $\pm 1.8 \times 10^{-6}$	0.779 $\pm 4.6 \times 10^{-4}$	0.0507 $\pm 8.1 \times 10^{-5}$
0.85	1.50×10^{-3} $\pm 3.4 \times 10^{-6}$	4.32×10^{-3} $\pm 2.9 \times 10^{-5}$	1.36×10^{-3} $\pm 4.1 \times 10^{-6}$	1.36×10^{-3} $\pm 4.4 \times 10^{-6}$	1.49×10^{-3} $\pm 3.2 \times 10^{-6}$	0.837 $\pm 7.7 \times 10^{-4}$	0.0674 $\pm 6.6 \times 10^{-5}$
0.9	2.17×10^{-3} $\pm 2.1 \times 10^{-6}$	9.85×10^{-3} $\pm 2.3 \times 10^{-5}$	2.05×10^{-3} $\pm 2.6 \times 10^{-6}$	2.05×10^{-3} $\pm 2.6 \times 10^{-6}$	2.17×10^{-3} $\pm 1.9 \times 10^{-6}$	0.894 $\pm 1.7 \times 10^{-4}$	0.0993 $\pm 6.3 \times 10^{-5}$
0.95	4.23×10^{-3} $\pm 1.3 \times 10^{-5}$	4.05×10^{-2} $\pm 3.3 \times 10^{-4}$	4.12×10^{-3} $\pm 1.3 \times 10^{-5}$	4.12×10^{-3} $\pm 1.4 \times 10^{-5}$	4.23×10^{-3} $\pm 1.4 \times 10^{-5}$	0.949 $\pm 2.6 \times 10^{-4}$	0.203 $\pm 6.2 \times 10^{-4}$
0.98	1.01×10^{-2} $\pm 4.1 \times 10^{-5}$	0.258 $\pm 2.9 \times 10^{-3}$	1.00×10^{-2} $\pm 4.1 \times 10^{-5}$	1.02×10^{-2} $\pm 4.5 \times 10^{-5}$	1.01×10^{-2} $\pm 4.1 \times 10^{-5}$	0.981 $\pm 1.5 \times 10^{-4}$	0.491 $\pm 3.4 \times 10^{-3}$

Table 1: Comparison of the ASE's for the different predictions in the $M/M/100$ model for alternative values of ρ.

ρ	ASE(LES)	ASE(EA)	ASE(WA)	ASE(WA-run)	ASE(EXP)	r	$\mathbb{E}[W \mid W > 0]$
0.7	8.97×10^{-4} $\pm 2.4 \times 10^{-6}$	1.35×10^{-3} $\pm 5.7 \times 10^{-6}$	7.82×10^{-4} $\pm 2.6 \times 10^{-6}$	7.93×10^{-4} $\pm 2.9 \times 10^{-6}$	8.96×10^{-4} $\pm 3.1 \times 10^{-6}$	0.687 $\pm 4.8 \times 10^{-4}$	0.0348 $\pm 5.6 \times 10^{-5}$
0.75	8.19×10^{-4} $\pm 2.1 \times 10^{-6}$	1.21×10^{-3} $\pm 4.8 \times 10^{-6}$	6.72×10^{-4} $\pm 2.1 \times 10^{-6}$	6.62×10^{-4} $\pm 2.2 \times 10^{-6}$	8.09×10^{-4} $\pm 2.0 \times 10^{-6}$	0.688 $\pm 9.0 \times 10^{-4}$	0.0385 $\pm 5.1 \times 10^{-5}$
0.8	1.03×10^{-3} $\pm 4.1 \times 10^{-7}$	2.18×10^{-3} $\pm 3.2 \times 10^{-6}$	9.09×10^{-4} $\pm 4.1 \times 10^{-7}$	9.10×10^{-4} $\pm 3.5 \times 10^{-7}$	1.02×10^{-3} $\pm 4.1 \times 10^{-7}$	0.782 $\pm 3.7 \times 10^{-4}$	0.0458 $\pm 1.3 \times 10^{-5}$
0.85	1.32×10^{-3} $\pm 1.8 \times 10^{-6}$	3.66×10^{-3} $\mp 2.0 \times 10^{-5}$	1.20×10^{-3} $\pm 2.1 \times 10^{-6}$	1.20×10^{-3} $\pm 2.4 \times 10^{-6}$	1.31×10^{-3} $\pm 1.8 \times 10^{-6}$	0.831 $\pm 8.4 \times 10^{-4}$	0.0619 $\pm 1.1 \times 10^{-4}$
0.9	1.85×10^{-3} $\pm 1.7 \times 10^{-6}$	8.46×10^{-3} $\pm 1.2 \times 10^{-5}$	1.74×10^{-3} $\pm 1.4 \times 10^{-5}$	1.74×10^{-3} $\pm 1.4 \times 10^{-6}$	1.85×10^{-3} $\pm 2.0 \times 10^{-6}$	0.896 $\pm 3.3 \times 10^{-5}$	0.0920 $\pm 1.7 \times 10^{-4}$
0.95	3.65×10^{-3} $\pm 5.3 \times 10^{-6}$	4.00×10^{-2} $\pm 1.7 \times 10^{-4}$	3.57×10^{-3} $\pm 6.1 \times 10^{-6}$	3.57×10^{-3} $\pm 5.9 \times 10^{-6}$	3.65×10^{-3} $\pm 5.4 \times 10^{-6}$	0.955 $\pm 1.3 \times 10^{-4}$	0.192 $\pm 6.8 \times 10^{-5}$
0.98	9.42×10^{-3} $\pm 5.5 \times 10^{-6}$	0.245 $\pm 4.1 \times 10^{-4}$	9.34×10^{-3} $\pm 5.4 \times 10^{-6}$	9.36×10^{-3} $\pm 1.7 \times 10^{-5}$	9.42×10^{-3} $\pm 5.5 \times 10^{-6}$	0.981 $\pm 1.2 \times 10^{-5}$	0.497 $\pm 9.8 \times 10^{-4}$

Table 2: Comparison of the ASE's for the different predictions in the $M/LN(1,1)/100$ model for alternative values of ρ, i.e., where $\mathbb{E}[S] = \mathrm{Var}[S] = 1$.

ρ	ASE(LES)	ASE(EA)	ASE(WA)	ASE(WA-run)	ASE(EXP)	r	$\mathbb{E}[W\|W>0]$
0.7	6.35×10^{-4} $\pm1.9\times10^{-6}$	6.10×10^{-4} $\pm4.1\times10^{-6}$	4.94×10^{-4} $\pm1.6\times10^{-6}$	4.69×10^{-4} $\pm2.5\times10^{-6}$	6.16×10^{-4} $\pm1.8\times10^{-6}$	0.530 $\pm4.0\times10^{-3}$	0.027 $\pm1.0\times10^{-4}$
0.75	6.98×10^{-4} $\pm3.0\times10^{-6}$	7.82×10^{-4} $\pm4.1\times10^{-6}$	5.66×10^{-4} $\pm3.0\times10^{-6}$	5.35×10^{-4} $\pm3.1\times10^{-6}$	6.88×10^{-4} $\pm2.9\times10^{-6}$	0.597 $\pm1.7\times10^{-3}$	0.0317 $\pm2.7\times10^{-5}$
0.8	8.57×10^{-4} $\pm1.8\times10^{-6}$	1.205×10^{-3} $\pm3.3\times10^{-6}$	7.28×10^{-4} $\pm1.5\times10^{-6}$	6.98×10^{-4} $\pm1.7\times10^{-6}$	8.50×10^{-4} $\pm1.9\times10^{-6}$	0.674 $\pm4.0\times10^{-4}$	0.0386 $\pm6.2\times10^{-5}$
0.85	1.01×10^{-3} $\pm1.0\times10^{-6}$	1.71×10^{-3} $\pm2.7\times10^{-6}$	8.93×10^{-4} $\pm6.9\times10^{-7}$	8.56×10^{-4} $\pm6.7\times10^{-7}$	1.00×10^{-3} $\pm1.0\times10^{-6}$	0.727 $\pm3.0\times10^{-4}$	0.0472 $\pm5.9\times10^{-5}$
0.9	1.28×10^{-3} $\pm1.5\times10^{-6}$	3.07×10^{-3} $\pm7.4\times10^{-6}$	1.18×10^{-3} $\pm1.9\times10^{-6}$	1.14×10^{-3} $\pm2.1\times10^{-6}$	1.28×10^{-3} $\pm1.7\times10^{-6}$	0.804 $\pm2.5\times10^{-4}$	0.0635 $\pm8.1\times10^{-5}$
0.95	1.99×10^{-3} $\pm1.2\times10^{-6}$	1.02×10^{-2} $\pm2.0\times10^{-5}$	1.92×10^{-3} $\pm1.2\times10^{-6}$	1.90×10^{-3} $\pm2.0\times10^{-6}$	1.99×10^{-3} $\pm1.3\times10^{-6}$	0.907 $\pm1.7\times10^{-4}$	0.113 $\pm9.1\times10^{-5}$
0.98	3.78×10^{-3} $\pm3.11\times10^{-6}$	6.36×10^{-2} $\pm1.4\times10^{-4}$	3.73×10^{-3} $\pm3.1\times10^{-6}$	3.72×10^{-3} $\pm3.5\times10^{-6}$	3.77×10^{-3} $\pm3.3\times10^{-6}$	0.971 $\pm5.8\times10^{-5}$	0.259 $\pm2.9\times10^{-4}$

Table 3: Comparison of the ASE's for the different predictions in the $M/D/100$ model for alternative values of ρ, i.e., where $\mathbb{E}[S]=1$.

ρ	ASE(LES)	ASE(EA)	ASE(WA)	ASE(WA-run)	ASE(EXP)	r	$\mathbb{E}[W\|W>0]$
0.7	8.16×10^{-4} $\pm1.8\times10^{-5}$	1.29×10^{-3} $\pm7.6\times10^{-5}$	6.90×10^{-4} $\pm2.0\times10^{-5}$	7.41×10^{-4} $\pm3.8\times10^{-5}$	8.10×10^{-4} $\pm1.9\times10^{-5}$	0.704 $\pm1.5\times10^{-2}$	0.0329 $\pm2.0\times10^{-4}$
0.75	8.25×10^{-4} $\pm3.8\times10^{-6}$	1.10×10^{-3} $\pm5.2\times10^{-6}$	6.91×10^{-4} $\pm3.3\times10^{-6}$	6.78×10^{-4} $\pm3.2\times10^{-6}$	8.16×10^{-4} $\pm4.1\times10^{-6}$	0.654 $\pm4.0\times10^{-4}$	0.0351 $\pm6.3\times10^{-5}$
0.8	9.76×10^{-4} $\pm2.1\times10^{-6}$	1.65×10^{-3} $\pm3.8\times10^{-6}$	8.40×10^{-4} $\pm2.0\times10^{-6}$	8.24×10^{-4} $\pm2.1\times10^{-6}$	9.70×10^{-4} $\pm2.1\times10^{-6}$	0.728 $\pm2.4\times10^{-4}$	0.0440 $\pm2.7\times10^{-5}$
0.85	1.21×10^{-3} $\pm3.1\times10^{-6}$	2.82×10^{-3} $\pm2.1\times10^{-5}$	1.09×10^{-3} $\pm3.0\times10^{-6}$	1.08×10^{-3} $\pm3.4\times10^{-6}$	1.21×10^{-3} $\pm3.1\times10^{-6}$	0.799 $\pm1.0\times10^{-3}$	0.0562 $\pm1.1\times10^{-4}$
0.9	1.66×10^{-3} $\pm1.0\times10^{-6}$	6.00×10^{-3} $\pm4.8\times10^{-6}$	1.55×10^{-3} $\pm1.3\times10^{-6}$	1.55×10^{-3} $\pm1.1\times10^{-6}$	1.66×10^{-3} $\pm1.1\times10^{-6}$	0.869 $\pm5.6\times10^{-5}$	0.0815 $\pm3.3\times10^{-5}$
0.95	3.00×10^{-3} $\pm4.6\times10^{-6}$	2.36×10^{-2} $\pm2.3\times10^{-4}$	2.91×10^{-3} $\pm5.6\times10^{-6}$	2.91×10^{-3} $\pm5.9\times10^{-6}$	3.00×10^{-3} $\pm4.7\times10^{-6}$	0.938 $\pm5.0\times10^{-4}$	0.158 $\pm2.4\times10^{-4}$
0.98	6.32×10^{-3} $\pm3.9\times10^{-6}$	0.114 $\pm2.7\times10^{-4}$	6.24×10^{-3} $\pm4.2\times10^{-6}$	6.24×10^{-3} $\pm7.4\times10^{-6}$	6.32×10^{-3} $\pm3.9\times10^{-6}$	0.973 $\pm5.6\times10^{-5}$	0.363 $\pm7.7\times10^{-4}$

Table 4: Comparison of the ASE's for the different predictions in the $M/E_2/100$ model for alternative values of ρ, i.e., where $\mathbb{E}[S]=1$.

ρ	ASE(LES)	ASE(EA)	ASE(WA)	ASE(WA-run)	ASE(EXP)	r	$\mathbb{E}[W\|W>0]$
1.1	4.26×10^{-3} $\pm 1.4 \times 10^{-5}$	1.51×10^{-2} $\pm 1.0 \times 10^{-4}$	4.00×10^{-3} $\pm 1.3 \times 10^{-5}$	3.96×10^{-3} $\pm 1.3 \times 10^{-5}$	4.26×10^{-3} $\pm 1.4 \times 10^{-5}$	0.862 $\pm 8.2 \times 10^{-4}$	0.218 $\pm 8.3 \times 10^{-4}$
1.3	9.4×10^{-3} $\pm 2.5 \times 10^{-5}$	2.00×10^{-2} $\pm 1.2 \times 10^{-4}$	8.33×10^{-3} $\pm 2.5 \times 10^{-5}$	8.33×10^{-3} $\pm 2.5 \times 10^{-5}$	9.44×10^{-3} $\pm 2.5 \times 10^{-5}$	0.766 $\pm 9.5 \times 10^{-4}$	0.530 $\pm 7.7 \times 10^{-4}$
1.5	1.35×10^{-2} $\pm 3.1 \times 10^{-5}$	2.02×10^{-2} $\pm 1.1 \times 10^{-4}$	1.13×10^{-2} $\pm 2.2 \times 10^{-5}$	1.13×10^{-2} $\pm 2.2 \times 10^{-5}$	1.35×10^{-2} $\pm 3.2 \times 10^{-5}$	0.666 $\pm 2.1 \times 10^{-3}$	0.816 $\pm 1.1 \times 10^{-3}$
1.7	1.67×10^{-2} $\pm 3.2 \times 10^{-5}$	2.02×10^{-2} $\pm 1.6 \times 10^{-4}$	1.33×10^{-2} $\pm 3.7 \times 10^{-5}$	1.33×10^{-2} $\pm 3.7 \times 10^{-5}$	1.67×10^{-2} $\pm 3.3 \times 10^{-5}$	0.588 $\pm 3.1 \times 10^{-3}$	1.07 $\pm 6.9 \times 10^{-4}$
2	0.0202 $\pm 1.2 \times 10^{-4}$	0.0200 $\pm 2.1 \times 10^{-4}$	0.0151 $\pm 1.1 \times 10^{-4}$	0.0151 $\pm 1.1 \times 10^{-4}$	0.0202 $\pm 1.2 \times 10^{-4}$	0.496 $\pm 3.9 \times 10^{-3}$	1.39 $\pm 1.1 \times 10^{-3}$
2.2	0.022 $\pm 1.2 \times 10^{-4}$	0.0201 $\pm 1.4 \times 10^{-4}$	0.0160 $\pm 9.2 \times 10^{-5}$	0.0160 $\pm 9.3 \times 10^{-5}$	0.0220 $\pm 1.2 \times 10^{-4}$	0.455 $\pm 23.0 \times 10^{-3}$	1.58 $\pm 1.0 \times 10^{-3}$
2.5	0.0241 $\pm 2.4 \times 10^{-4}$	0.0203 $\pm 2.0 \times 10^{-4}$	0.0169 $\pm 1.7 \times 10^{-4}$	0.0169 $\pm 1.7 \times 10^{-4}$	0.0241 $\pm 2.4 \times 10^{-4}$	0.399 $\pm 1.9 \times 10^{-3}$	1.84 $\pm 8.2 \times 10^{-4}$

Table 5: Comparison of the ASE's for the different predictions in the $M/M/100+M$ model for alternative values of ρ, i.e., where $\mathbb{E}[S]=1$ and $\theta=0.5$.

ρ	ASE(LES)	ASE(EA)	ASE(WA)	ASE(WA-run)	ASE(EXP)	r	$\mathbb{E}[W\|W>0]$
1.1	0.0155 ±4.4E-05	0.0294 ±0.00028	0.0143 ±4.0E-05	0.0134 ±3.8E-05	0.0155 ±4.3E-05	0.738 ±0.0026	0.923 ±0.00093
1.3	0.0173 ±4.4E-05	0.0155 ±7.4E-05	0.0141 ±3.7E-05	0.0125 ±3.8E-05	0.0172 ±4.4E-05	0.445 ±0.0023	1.26 ±0.00055
1.5	0.0174 ±5.2E-05	0.0128 ±5.1E-05	0.0130 ±3.7E-05	0.0115 ±3.3E-05	0.0174 ±5.2E-05	0.320 ±0.0027	1.46 ±0.00045
1.7	0.0175 ±7.7E-05	0.0116 ±4.7E-05	0.0123 ±5.2E-05	0.0109 ±4.2E-05	0.0175 ±7.7E-05	0.248 ±0.0028	1.61 ±0.00045
2	0.0174 ±7.0E-05	0.0106 ±4.5E-05	0.0113 ±4.1E-05	0.0103 ±3.8E-05	0.0174 ±7.0E-05	0.183 ±0.0036	1.79 ±0.00047
2.2	0.0173 ±6.8E-05	0.0103 ±4.0E-05	0.0109 ±3.9E-05	0.0100 ±3.7E-05	0.0173 ±6.8E-05	0.159 ±0.0024	1.89 ±0.00038
2.5	0.0173 ±8.2E-05	0.00992 ±5.6E-05	0.0105 ±5.3E-05	0.00977 ±5.2E-05	0.0173 ±8.2E-05	0.127 ±0.0023	2.02 ±0.00021

Table 6: Comparison of the ASE's for the different predictions in the $M/M/100+LN(2,1)$ model for alternative values of ρ, for $\mathbb{E}[T]=2$ and $\mathrm{Var}[T]=1$ where T is time to abandon.

ρ	ASE(LES)	ASE(EA)	ASE(WA)	ASE(WA-run)	ASE(EXP)	r	$\mathbb{E}[W\|W>0]$
1.1	0.00905 ±2.2E-05	0.0306 ±0.00020	0.00849 ±2.1E-05	0.00839 ±2.3E-05	0.00906 ±2.2E-05	0.854 ±0.00084	0.488 ±0.0010
1.3	0.0148 ±5.6E-05	0.0212 ±0.00013	0.0125 ±4.6E-05	0.0122 ±4.5E-05	0.0148 ±5.5E-05	0.653 ±0.0021	0.908 ±0.00061
1.5	0.0176 ±5.9E-05	0.0185 ±9.31E-05	0.0138 ±4.5E-05	0.0134 ±4.4E-05	0.0176 ±5.9E-05	0.526 ±0.0023	1.19 ±0.00081
1.7	0.0193 ±9.1E-05	0.0172 ±8.5E-05	0.0142 ±6.5E-05	0.0139 ±6.2E-05	0.0193 ±9.1E-05	0.439 ±0.0017	1.41 ±0.00052
2	0.0209 ±9.0E-05	0.0159 ±5.9E-05	0.0144 ±5.3E-05	0.0141 ±4.9E-05	0.0209 ±8.9E-05	0.347 ±0.0022	1.68 ±0.00044
2.2	0.0215 ±0.00011	0.0154 ±0.00012	0.0144 ±7.9E-05	0.0140 ±8.5E-05	0.0215 ±0.00011	0.304 ±0.0037	1.83 ±0.00052
2.5	0.0224 ±9.5E-05	0.0151 ±8.9E-05	0.0144 ±5.9E-05	0.0141 ±6.4E-05	0.0224 ±9.5E-05	0.259 ±0.0040	2.03 ±0.00067

Table 7: Comparison of the ASE's for the different predictions in the $M/M/100 + E_2$ model for alternative values of ρ, i.e., where $\mathbb{E}[S] = \text{Var}[S] = 1$ and $\theta = 0.5$.

ρ	ASE(LES)	ASE(EA)	ASE(WA)	ASE(WA-run)	ASE(EXP)	r	$\mathbb{E}[W\|W>0]$
1.1	0.00187 ±4.7E-06	0.00387 ±1.3E-05	0.00174 ±4.3E-06	0.00165 ±4.1E-06	0.00187 ±4.69E-06	0.767 ±0.00043	0.0937 ±0.00020
1.3	0.00318 ±6.0E-06	0.00605 ±2.1E-05	0.00277 ±5.5E-06	0.00276 ±5.5E-06	0.00318 ±6.0E-06	0.743 ±0.00094	0.174 ±0.00020
1.5	0.00448 ±9.4E-06	0.00659 ±1.7E-05	0.00372 ±6.8E-06	0.00372 ±6.8E-06	0.00448 ±9.4E-06	0.665 ±0.00091	0.265 ±0.00019
1.7	0.00554 ±1.5E-05	0.00674 ±1.6E-05	0.00440 ±1.02E-05	0.00440 ±1.0E-05	0.00554 ±1.5E-05	0.593 ±0.0013	0.347 ±0.00022
2	0.00681 ±1.9E-05	0.00690 ±3.3E-05	0.00513 ±1.7E-05	0.00513 ±1.6E-05	0.00680 ±1.9E-05	0.511 ±0.0015	0.457 ±0.00024
2.2	0.00750 ±1.5E-05	0.00706 ±2.7E-05	0.00551 ±1.1E-05	0.00551 ±1.1E-05	0.007498 ±1.5E-05	0.472 ±0.0020	0.523 ±0.00032
2.5	0.00840 ±2.7E-05	0.00725 ±3.7E-05	0.00597 ±2.2E-05	0.00597 ±2.2E-05	0.00839 ±2.7E-05	0.424 ±0.0024	0.613 ±0.00023

Table 8: Comparison of the ASE's for the different predictions in the $M/M/100 + H_2$ model for alternative values of ρ, i.e., where $\mathbb{E}[S] = \text{Var}[S] = 1$ and $\text{Var}[T] = 4$ where T is time to abandon.

ρ	ASE(LES)	ASE(EA)	ASE(WA)	ASE(WA-run)	ASE(EXP)	r	$\mathbb{E}[W\|W>0]$
1.1	0.00816 ±2.2E-05	0.0234 ±0.00017	0.00762 ±2.1E-05	0.00745 ±2.1E-05	0.00816 ±2.3E-05	0.827 ±0.0014	0.449 ±0.00062
1.3	0.0123 ±4.5E-05	0.0164 ±9.3E-05	0.0103 ±3.4E-05	0.0100 ±3.1E-05	0.0123 ±4.5E-05	0.627 ±0.0022	0.766 ±0.00060
1.5	0.0146 ±4.3E-05	0.0152 ±9.8E-05	0.0114 ±3.41E-05	0.0111 ±3.7E-05	0.0146 ±4.3E-05	0.520 ±0.0031	0.991 ±0.00046
1.7	0.0164 ±7.5E-05	0.0149 ±6.8E-05	0.0121 ±5.3E-05	0.0118 ±5.2E-05	0.0164 ±7.5E-05	0.450 ±0.0015	1.17 ±0.00055
2	0.0184 ±8.3E-05	0.0148 ±5.0E-05	0.0129 ±4.9E-05	0.0127 ±4.4E-05	0.0184 ±8.3E-05	0.382 ±0.0024	1.42 ±0.00064
2.2	0.0194 ±9.8E-05	0.0149 ±4.6E-05	0.0132 ±5.7E-05	0.0131 ±5.2E-05	0.0194 ±9.8-05	0.349 ± 0.0020	1.56 ±0.00052
2.5	0.0208 ±9.9E-05	0.0151 ±8.3E-05	0.0137 ±5.4E-05	0.0136 ±5.5E-05	0.0208 ±9.9E-05	0.310 ±0.0042	1.75 ±0.00071

Table 9: Comparison of the ASE's for the different predictions in the $M/M/100 + LN(2,4)$ model for alternative values of ρ, i.e., where $\mathbb{E}[S] = \text{Var}[S] = 1$, $\mathbb{E}[T] = 2$, $\text{Var}[T] = 4$ where T is time to abandon.

ρ	ASE(LES)	ASE(EA)	ASE(WA)	ASE(WA-run)	ASE(EXP)	r	$\mathbb{E}[W\|W>0]$
1.1	0.0126 ±3.6E-05	0.0101 ±0.00014	0.0115 ±3.3E-05	0.00864 ±6.5E-05	0.0125 ±3.7E-05	0.377 ±0.0081	1.91 ±0.00042
1.3	0.00241 ±1.7E-05	0.00121 ±9.6E-06	0.00191 ±1.3E-05	0.00120 ±9.6E-06	0.00237 ±1.7E-05	0.00326 ±0.0014	1.97 ±7.8E-05
1.5	0.00100 ±2.9E-06	0.000500 ±1.2E-06	0.000723 ±2.1E-06	0.000500 ±1.2E-06	0.000975 ±3.1E-06	0.0000877 ±0.00091	1.98 ±3.4E-05
1.7	0.000608 ±1.5E-06	0.000304 ±7.3E-07	0.000409 ±1.0E-06	0.000304 ±7.3E-07	0.000588 ±1.4E-06	-0.000303 ±0.00065	1.99 ±1.12E-05
2	0.000399 ±5.8E-07	0.000200 ±4.0E-07	0.000250 ±4.5E-07	0.000200 ±4.0E-07	0.000384 ±5.2E-07	0.000341 ±0.00034	1.99 ±5.9E-06
2.2	0.000338 ±3.4E-07	0.000169 ±3.0E-07	0.000204 ±3.6E-07	0.000169 ±3.06E-07	0.000325 ±4.5E-07	-0.000304 ±0.00051	2.00 ±8.8E-06
2.5	0.000289 ±3.6E-07	0.000144 ±3.0E-07	0.000168 ±2.2E-07	0.000144 ±3.0E-07	0.000278 ±3.4E-07	0.000192 ±0.00042	2.00 ±8.0E-06

Table 10: Comparison of the ASE's for the different predictions in the $M/M/100 + D$ model for alternative values of ρ, i.e., where $\mathbb{E}[S] = \text{Var}[S] = 1$, and $\mathbb{E}[T] = 2$ where T is time to abandon.

3.2. Tables with Simulation Results for Two-Class Model with Priorities

ρ_L	ρ_H	ASE(LES)	ASE(EA)	ASE(WA)	ASE(WA-run)	ASE(EXP)	r	$\mathbb{E}[W\|W>0]$
0.35	0.5	0.0164	0.0234	0.0139	0.0139	0.0163	0.659	0.132
		±0.00011	±0.00058	±0.00014	±0.00015	±0.00011	±0.0071	±0.00071
0.375	0.5	0.0189	0.0312	0.0164	0.0164	0.0188	0.707	0.158
		±0.00013	±0.00057	±0.00013	±0.00014	±0.00013	±0.0040	±0.00074
0.4	0.5	0.0227	0.0478	0.0204	0.0203	0.0227	0.771	0.200
		±0.00014	±0.00081	±0.00015	±0.00015	±0.00015	±0.0025	±0.0013
0.425	0.5	0.0281	0.0808	0.0259	0.0259	0.0280	0.832	0.264
		±0.00022	±0.0021	±0.00024	±0.00025 ±	0.00022	±0.0031	±0.0014
0.45	0.5	0.0393	0.176	0.0373	0.0373	0.0393	0.892	0.399
		±0.00047	±0.0049	±0.00048	±0.00048	±0.00047	±0.0019	±0.0043
0.475	0.5	0.0703	0.649	0.0685	0.0685	0.0703	0.946	0.789
		±0.00098	±0.026	±0.00096	±0.00097	±0.00098	±0.0018	±0.0086
0.49	0.5	0.155	3.219	0.154	0.155	0.155	0.975	1.85
	±	0.0053	±0.29	±0.0054	±0.0048	±0.0053	±0.0013	±0.059

Table 11: Comparison of the ASE's for the different predictions in the two-class $M/M/100$ model with priorities for alternative values of ρ_L and ρ_H.

ρ_L	ρ_H	ASE(LES)	ASE(EA)	ASE(WA)	ASE(WA-run)	ASE(EXP)	r	$\mathbb{E}[W\|W>0]$
0.35	0.5	0.0134 ±0.00013	0.0193 ±0.00022	0.0114 ±0.00011	0.0114 ±0.00011	0.0133 ±0.00013	0.664 ±0.0025	0.120 ±0.00034
0.375	0.5	0.0156 ±0.00013	0.0276 ±0.00048	0.0137 ±0.00012	0.0137 ±0.00012	0.0155 ±0.00013	0.729 ±0.0035	0.147 ±0.00073
0.4	0.5	0.0186 ±0.00015	0.0414 ±0.00064	0.0167 ±0.00014	0.0167 ±0.00014	0.0185 ±0.00015	0.784 ±0.0022	0.184 ±0.00097
0.425	0.5	0.0232 ±0.00023	0.0724 ±0.0015	0.0215 ±0.00023	0.0215 ±0.00023	0.0232 ±0.00023	0.846 ±0.0021	0.246 ±0.0016
0.45	0.5	0.03305 ±0.00046	0.164 ±0.0073	0.0315 ±0.00050	0.0315 ±0.00049	0.0330 ±0.00046	0.902 ±0.0030	0.377 ±0.0048
0.475	0.5	0.0620 ±0.0010	0.618 ±0.033	0.0605 ±0.0011	0.0605 ±0.0011	0.0620 ±0.0010	0.950 ±0.0020	0.768 ±0.010
0.49	0.5	0.146 ±0.0028	3.62 ±0.19	0.145 ±0.0028	0.146 ±0.0022	0.146 ±0.0028	0.979 ±0.00087	1.87 ±0.033

Table 12: Comparison of the ASE's for the different predictions in the two-class $M/LN(1,1)/100$ model with priorities for alternative values of ρ_L and ρ_H.

ρ_L	ρ_H	ASE(LES)	ASE(EA)	ASE(WA)	ASE(WA-run)	ASE(EXP)	r	$\mathbb{E}[W\|W>0]$
0.55	0.5	0.0266 ±5.3E-05	0.0477 ±0.00020	0.0247 ±4.9E-05	0.0232 ±4.9E-05	0.0266 ±5.3E-05	0.728 ±0.00090	0.300 ±0.00057
0.65	0.5	0.0410 ±0.00010	0.0722 ±0.00030	0.0357 ±9.9E-05	0.0355 ±0.00010	0.0410 ±0.00010	0.7206 ±0.00076	0.537 ±0.00075
0.75	0.5	0.0557 ±0.00013	0.0784 ±0.00030	0.0462 ±0.00012	0.0461 ±0.00012	0.0557 ±0.00013	0.647 ±0.0011	0.811 ±0.0011
0.85	0.5	0.0674 ±0.00020	0.0787 ±0.00041	0.0533 ±0.00016	0.0533 ±0.00016	0.0673 ±0.00021	0.573 ±0.0024	1.06 ±0.00083
1	0.5	0.0808 ± 0.00018	0.0787 ±0.00024	0.0604 ±0.00013	0.0604 ± 0.00013	0.0808 ±0.00018	0.486 ±0.0014	1.38 ±0.0013
1.1	0.5	0.0873 ±0.00035	0.0786 ±0.00036	0.0634 ±0.00022	0.0634 ±0.00022	0.0873 ±0.00035	0.444 ±0.0027	1.57 ±0.0010
1.25	0.5	0.0959 ±0.00039	0.0786 ±0.00051	0.0670 ±0.00032	0.0670 ±0.00033	0.0959 ±0.00039	0.387 ±0.0023	1.832 ±0.0018

Table 13: Comparison of the ASE's for the different predictions in the two-class $M/M/100+M$ model with priorities for alternative values of ρ_L and ρ_H, with $\theta = 0.5$.

ρ_L	ρ_H	ASE(LES)	ASE(EA)	ASE(WA)	ASE(WA-run)	ASE(EXP)	r	$\mathbb{E}[W\|W>0]$
0.55	0.5	0.0603 $\pm$ 0.00017	0.122 $\pm$0.00043	0.0559 $\pm$ 0.00015	0.0531 $\pm$0.00011	0.0602 $\pm$ 0.00017	0.756 $\pm$0.0013	0.852 $\pm$0.0020
0.65	0.5	0.0707 $\pm$ 0.00025	0.0675 $\pm$0.00041	0.0583 $\pm$0.00021	0.0526 $\pm$0.00021	0.0706 $\pm$0.00025	0.476 $\pm$0.0029	1.24 $\pm$0.00064
0.75	0.5	0.0718 $\pm$0.00023	0.0534 $\pm$0.00021	0.0542 $\pm$0.00018	0.0480 $\pm$0.00016	0.0718 $\pm$0.00023	0.322 $\pm$ 0.0022	1.460 $\pm$0.00057
0.85	0.5	0.0719 $\pm$ 0.00016	0.0482 $\pm$0.00012	0.0508 $\pm$ 0.00011	0.0454 $\pm$0.00010	0.0718 $\pm$0.00016	0.246 $\pm$0.0012	1.61 $\pm$0.00052
1	0.5	0.0717 $\pm$ 0.00033	0.0443 $\pm$0.00017	0.0473 $\pm$0.00021	0.0429 $\pm$0.00017	0.0716 $\pm$0.00033	0.179 $\pm$0.0013	1.80 $\pm$0.00054
1.1	0.5	0.0711 $\pm$ 0.00020	0.0426 $\pm$0.00018	0.0454 $\pm$0.00014	0.0417 $\pm$0.00015	0.0710 $\pm$0.00020	0.152 $\pm$0.0028	1.90 $\pm$0.00097
1.25	0.5	0.0709 $\pm$0.00030	0.0411 $\pm$0.00021	0.0435 $\pm$0.00019	0.0405 $\pm$0.000206	0.0709 $\pm$0.00030	0.124 $\pm$0.0031	2.03 $\pm$0.00070

Table 14: Comparison of the ASE's for the different predictions in the two-class $M/M/100 + LN(2,1)$ model with priorities for alternative values of ρ_L and ρ_H, with $\mathbb{E}[T] = 2$ where T is time to abandon.

ρ_L	ρ_H	ASE(LES)	ASE(EA)	ASE(WA)	ASE(WA-run)	ASE(EXP)	r	$\mathbb{E}[W\|W>0]$
0.55	0.5	0.0425 $\pm$0.00013	0.0892 $\pm$0.00029	0.0395 $\pm$0.00012	0.0377 $\pm$0.00011	0.0425 $\pm$0.00013	0.766 $\pm$0.00068	0.521 $\pm$0.0011
0.65	0.5	0.0623 $\pm$ 0.00014	0.0876 $\pm$0.00044	0.0528 $\pm$0.00012	0.0516 $\pm$0.00011	0.0622 $\pm$0.00014	0.647 $\pm$0.0020	0.902 $\pm$0.0014
0.75	0.5	0.0734 $\pm$ 0.00019	0.0758 $\pm$ 0.00033	0.0577 $\pm$ 0.00015	0.0560 $\pm$ 0.00015	0.0733 $\pm$ 0.00019	0.516 $\pm$0.00212	1.19 $\pm$0.0013
0.85	0.5	0.0802 $\pm$0.00026	0.0702 $\pm$0.00028	0.0595 $\pm$0.00018	0.0577 $\pm$0.00018	0.0802 $\pm$0.00026	0.426 $\pm$0.0018	1.41 $\pm$0.0011
1	0.5	0.0861 $\pm$0.00055	0.0653 $\pm$0.00044	0.0598 $\pm$0.00037	0.0581 $\pm$ 0.00036	0.0860 $\pm$0.00055	0.337 $\pm$0.0028	1.68 $\pm$0.00066
1.1	0.5	0.0887 $\pm$0.00034	0.0633 $\pm$0.00033	0.0595 $\pm$0.00025	0.0580 $\pm$0.00026	0.0886 $\pm$0.00034	0.294 $\pm$0.0023	1.84 $\pm$0.00089
1.25	0.5	0.0912 $\pm$0.00046	0.0611 $\pm$0.00037	0.0589 $\pm$0.00030	0.0575 $\pm$0.00031	0.0911 $\pm$0.00046	0.247 $\pm$0.0040	2.03 $\pm$0.0013

Table 15: Comparison of the ASE's for the different predictions in the two-class $M/M/100 + E_2$ model with priorities for alternative values of ρ_L and ρ_H, with $\mathbb{E}[T] = 2$ where T is time to abandon.

ρ_L	ρ_H	ASE(LES)	ASE(EA)	ASE(WA)	ASE(WA-run)	ASE(EXP)	r	$\mathbb{E}[W \mid W > 0]$
0.55	0.5	0.0129	0.0140	0.0119	0.0102	0.0128	0.546	0.144
		±3.0E-05	±5.0E-05	±2.8E-05	±2.9E-05	±2.9E-05	±0.0010	±0.00018
0.65	0.5	0.0160	0.0188	0.0135	0.0129	0.0160	0.583	0.201
		±4.0E-05	±6.5E-05	±3.5E-05	±3.6E-05	±4.0E-05	±0.00083	±0.00025
0.75	0.5	0.0194	0.0227	0.0157	0.0156	0.0194	0.579	0.270
		± 2.4E-05	±5.0E-05	±2.3E-05	±2.4E-05	±2.4E-05	±0.00083	±0.00030
0.85	0.5	0.0228	0.0250	0.0179	0.0179	0.0228	0.548	0.345
		±5.8E-05	±6.2E-05	±5.0E-05	±5.0E-05	±5.8E-05	±0.00097	±0.00042
1	0.5	0.0272	0.0264	0.0205	0.0205	0.0272	0.485	0.454
		±8.7E-05	±0.00013	±7.99E-05	± 8.0E-05	±8.7E-05	±0.0013	±0.00036
1.1	0.5	0.0296	0.0268	0.0217	0.0217	0.0296	0.448	0.519
		±9.8E-05	±0.00011	±8.6E-05	±8.6E-05	±9.7E-05	±0.0011	±0.00038
1.25	0.5	0.0329	0.0275	0.0233	0.0233	0.0328	0.401	0.609
		±8.3E-05	±0.00011	±7.3E-05	±7.2E-05	±8.3E-05	±0.0020	±0.00059

Table 16: Comparison of the ASE's for the different predictions in the two-class $M/M/100 + H_2$ model with priorities for alternative values of ρ_L and ρ_H, with $\mathbb{E}[T] = 1$, $\text{Var}[T] = 4$ where T is time to abandon.

ρ_L	ρ_H	ASE(LES)	ASE(EA)	ASE(WA)	ASE(WA-run)	ASE(EXP)	r	$\mathbb{E}[W \mid W > 0]$
0.55	0.5	0.0603	0.122	0.0559	0.0531	0.0602	0.756	0.852
		± 0.00017	±0.00043	±0.00015	±0.00011	±0.00017	±0.0013	±0.0020
0.65	0.5	0.0707	0.0675	0.0583	0.0526	0.0706	0.476	1.24
		±0.00025	±0.00041	± 0.00021	±0.00021	±0.00025	±0.0029	±0.00064
0.75	0.5	0.0718	0.0534	0.0542	0.0480	0.0718	0.322	1.46
		±0.00023	±0.00021	± 0.00018	±0.00016	±0.00023	±0.0022	±0.00057
0.85	0.5	0.0719	0.0482	0.0508	0.0454	0.0718	0.246	1.61
		± 0.00016	±0.00012	± 0.00011	± 0.00011	±0.00016	± 0.0012	±0.00052
1	0.5	0.0717	0.0443	0.0473	0.0429	0.0716	0.179	1.801
		± 0.00033	±0.00017	±0.00021	±0.00017	±0.00033	±0.0013	±0.00054
1.1	0.5	0.0711	0.0426	0.0454	0.0417	0.0710	0.152	1.90
		± 0.00020	± 0.00018	±0.00014	±0.00015	± 0.00020	±0.0028	±0.00097
1.25	0.5	0.0709	0.0411	0.0435	0.0405	0.0709	0.124	2.03
		±0.00030	±0.00021	±0.00019	±0.00020	±0.00030	±0.0031	±0.00070

Table 17: Comparison of the ASE's for the different predictions in the two-class $M/M/100 + LN(2,4)$ model with priorities for alternative values of ρ_L and ρ_H, with $\mathbb{E}[T] = 2$, $\text{Var}[T] = 4$ where T is time to abandon.

ρ_L	ρ_H	ASE(LES)	ASE(EA)	ASE(WA)	ASE(WA-run)	ASE(EXP)	r	$\mathbb{E}[W \mid W > 0]$
0.55	0.5	0.0690 ±0.00019	0.129 ±0.0016	0.0639 ±0.00017	0.0601 ±0.00019	0.0689 ±0.00019	0.735 ±0.0035	1.66 ±0.0019
0.65	0.5	0.0278 ±0.00015	0.0170 ±0.00015	0.0226 ±0.00012	0.0167 ±0.00012	0.0277 ±0.00015	0.150 ±0.0052	1.92 ±0.00023
0.75	0.5	0.0135 ±5.02E-05	0.00767 ±3.5E-05	0.0102 ±4.0E-05	0.00767 ±3.5E-05	0.0134 ±5.0E-05	0.0124 ±0.0020	1.973 ±0.00011
0.85	0.5	0.00892 ±2.4E-05	0.00526 ±1.8E-0	0.00653 5 ±2.2E-05	0.00526 ±1.8E-05	0.00884 ±2.4E-05	0.000702 ±0.00089	1.99 ±7.2E-05
1	0.5	0.00661 ±2.1E-05	0.00411 ±1.7E-05	0.00474 ±1.9E-05	0.00411 ±1.7E-05	0.00655 ±2.1E-05	-0.000371 ± 0.00076	2.01 ±3.7E-05
1.1	0.5	0.00597 ±2.9E-05	0.00378 ±2.3E-05	0.00424 ±2.4E-05	0.00378 ±2.3E-05	0.00592 ±3.0E-05	-0.000246 ±0.00077	2.015 ±8.4E-05
1.25	0.5	0.0055 ±1.9E-05	0.00355 ±1.45E-05	0.003864 ±1.47E-05	0.00355 ±1.4E-05	0.00546 ±1.9E-05	0.000521 ±0.00083	2.02 ±6.4E-05

Table 18: Comparison of the ASE's for the different predictions in the two-class $M/M/100 + D$ model with priorities for alternative values of ρ_L and ρ_H, with $\mathbb{E}[T] = 2$ where T is time to abandon.

3.3. Tables with Simulation Results for Time-Varying Arrivals

γ	ρ_L	ASE(LES)	ASE(EA)	ASE(WA)	ASE(WA-run)	ASE(EXP)	r	$\mathbb{E}[W\|W>0]$
0	0.2	0.103 ± 0.00067	0.0775 ±0.00072	0.0737 ±0.00056	0.0729 ±0.00059	0.117 ±0.0098	0.283 ±0.0026	0.221 ±0.00051
0.0436	0.2	0.597 ±0.0076	0.818 ±0.016	0.556 ±0.0094	0.510 ±0.0074	0.815 ± 0.21	0.637 ±0.0034	0.723 ±0.0051
0.0873	0.2	0.566 ±0.0043	0.698 ±0.0097	0.499 ±0.0053	0.473 ±0.0042	0.682 ±0.17	0.593 ± 0.0037	0.682 ±0.0027
0.262	0.2	0.446 ±0.0030	0.417 ±0.0026	0.345 ± 0.0022	0.344 ±0.0022	0.450 ±0.0065	0.450 ± 0.0010	0.549 ±0.0019
0.524	0.2	0.320 ±0.0021	0.249 ±0.0021	0.230 ±0.0018	0.229 ±0.0018	0.350 ±0.041	0.321 ±0.0015	0.432 ±0.0012
1.571	0.2	0.147 ±0.00118	0.0981 ±0.00094	0.100 ±0.00086	0.0961 ±0.00087	0.153 ±0.010	0.183 ±0.0038	0.270 ±0.00054

Table 19: Comparison of the ASE's for the different predictions in the two-class $M_t/M/30$ model with priorities, sinusoidal arrivals with amplitude $\alpha = 0.3$ and varying frequency γ. We fix $\rho_H = 0.5$.

3.4. Smaller Systems

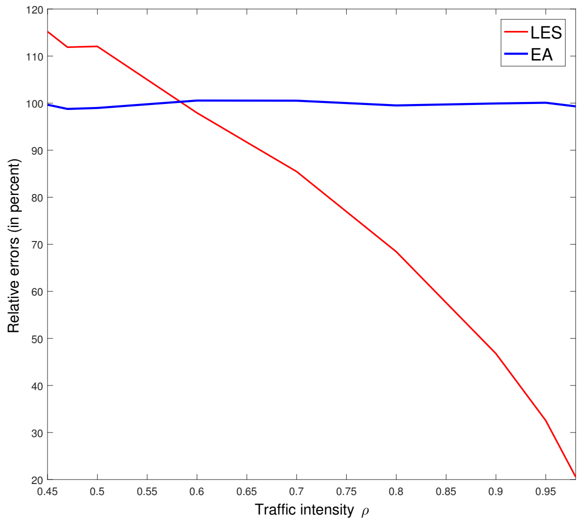

Figure 1: Relative errors in the single-class $M/M/30$ queue.

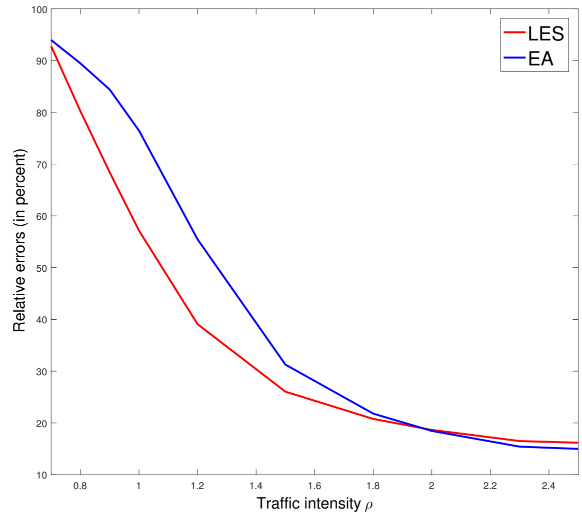

Figure 2: Relative errors in the single-class $M/M/30 + M$ queue.

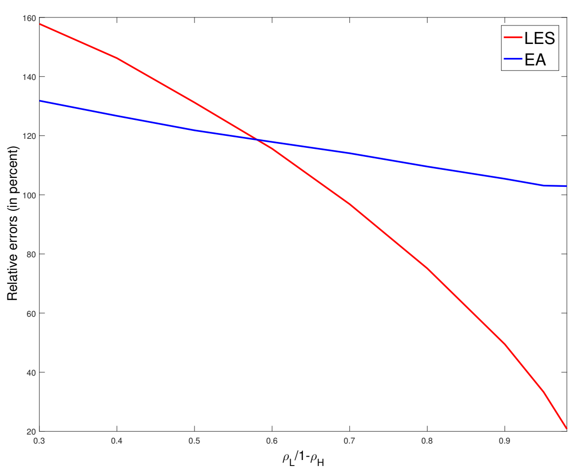

Figure 3: Relative errors in the two-class $M/M/30$ queue.

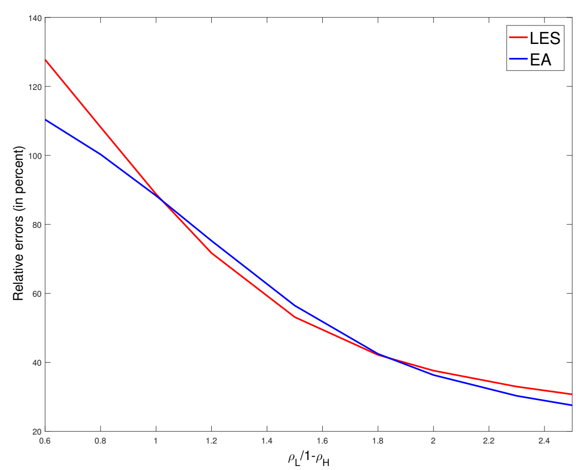

Figure 4: Relative errors in the two-class $M/M/30+M$ queue.

γ	ρ_L	ASE(LES)	ASE(EA)	ASE(WA)	ASE(WA-run)	ASE(EXP)	r	$\mathbb{E}[W\|W>0]$
0	0.3	0.0114 ±0.00047	0.0101 ± 0.00058	0.00895 ± 0.00041	0.008727 ±0.00043	0.0174 ±0.00073	0.411 ± 0.015	0.0857 ±0.001
0.0436	0.3	0.0543 ±0.0012	0.0747 ±0.0020	0.0461 ±0.0010	0.0456 ±0.0010	0.166 ±0.0044	0.640 ±0.0053	0.302 ±0.0041
0.0873	0.3	0.0528 ±0.00074	0.0718 ±0.0014	0.0448 ±0.00076	0.0447 ± 0.00074	0.159 ±0.0026	0.636 ±0.0039	0.2963 ±0.0021
0.262	0.3	0.0503 ± 0.00104	0.0633 ±0.0013	0.0417 ±0.00082	0.0417 ±0.00082	0.111 ±0.029	0.606 ±0.0037	0.273 ±0.0034
0.524	0.3	0.0433 ± 0.00088	0.0480 ±0.00097	0.0349 ±0.00076	0.0348 ±0.00076	0.101 ±0.0019	0.550 ±0.0039	0.231 ±0.0025
1.571	0.3	0.0233 ±0.00077	0.0194 ±0.00061	0.0177 ±0.00059	0.0170 ±0.00056	0.0349 ±0.0046	0.390 ±0.010	0.136 ±0.0014

Table 20: Comparison of the ASE's for the different predictions in the two-class $M_t/M/100+M$ model with priorities, sinusoidal arrivals with amplitude $\alpha = 0.3$ and varying frequency γ. We fix $\rho_H = 0.5$

4. Simulations Focusing on the Transient State

Run length	ASE(LES)	ASE(EA)	r	$1/\rho$	Error
200	0.0045	0.00544	0.736	0.714	0.0304
400	0.00570	0.0209	0.922	0.714	0.291
800	0.00256	0.00581	0.774	0.714	0.0841
1600	0.00536	0.00842	0.624	0.714	0.126
3200	0.00518	0.0113	0.763	0.714	0.0681
6400	0.00538	0.0103	0.729	0.714	0.0211
12800	0.00593	0.0103	0.697	0.714	0.0238
25600	0.00595	0.0107	0.718	0.714	0.00548
51200	0.00572	0.0102	0.721	0.714	0.00928
102400	0.00599	0.0104	0.715	0.714	0.00110

Table 21: Simulations in the $M/M/100+M$ model with $\theta=\mu=1$ and $\lambda=140$, for various lengths of the simulation run.

Run length	ASE(LES)	ASE(EA)	r	$1/\rho$	Error
1000	0.00001	0.000022	0.802	0.714	0.123
2000	0.00164	0.00913	0.965	0.714	0.351
4000	0.000787	0.00935	0.962	0.714	0.347
8000	0.00109	0.00651	0.909	0.714	0.273
16000	0.00127	0.00555	0.873	0.714	0.223
32000	0.00103	0.00279	0.810	0.714	0.134
64000	0.00121	0.00244	0.739	0.714	0.0343
128000	0.00113	0.00234	0.752	0.714	0.0525
256000	0.00116	0.00225	0.739	0.714	0.0343
512000	0.00114	0.00209	0.726	0.714	0.0170

Table 22: Simulations in the $M/M/500+M$ model with $\theta=\mu=1$ and $\rho=1.4$, for various lengths of the simulation run.

Run length	ASE(LES)	ASE(EA)	r	$1/\rho$	Error
2000	0.000042	0.000094	0.926	0.714	0.296
4000	0.000857	0.00794	0.979	0.714	0.370
8000	0.000824	0.00744	0.953	0.714	0.334
16000	0.000478	0.00471	0.947	0.714	0.326
32000	0.000448	0.00330	0.929	0.714	0.301
64000	0.00057	0.00260	0.885	0.714	0.239
128000	0.000609	0.00152	0.785	0.714	0.0993
256000	0.000525	0.00117	0.765	0.714	0.0709
512000	0.000561	0.00123	0.767	0.714	0.0746
1024000	0.000578	0.00120	0.756	0.714	0.0588

Table 23: Simulations in the $M/M/1000+M$ model with $\theta = \mu = 1$ and $\rho = 1.4$, for various lengths of the simulation run.

Run length	ASE(LES)	ASE(EA)	r	$1/\rho$	Error
2000	0.000166	0.000662	0.962	0.625	0.539
4000	0.00177	0.0138	0.989	0.625	0.582
8000	0.00235	0.0232	0.963	0.625	0.540
16000	0.00113	0.0152	0.965	0.625	0.544
32000	0.000811	0.00871	0.953	0.625	0.525
64000	0.000836	0.00424	0.897	0.625	0.436
128000	0.000779	0.00241	0.828	0.625	0.324
256000	0.000862	0.00173	0.720	0.625	0.152
512000	0.000803	0.00137	0.691	0.625	0.106
1024000	0.000772	0.00116	0.658	0.625	0.0523

Table 24: Simulations in the $M/M/1000+M$ model with $\theta = \mu = 1$ and $\rho = 1.6$, for various lengths of the simulation run.

Run length	ASE(LES)	ASE(EA)	r	$1/\rho$	Error
2000	0.000746	0.00237	0.922	0.556	0.660
4000	0.00483	0.0250	0.977	0.556	0.758
8000	0.00378	0.0310	0.968	0.556	0.742
16000	0.00207	0.0203	0.960	0.556	0.728
32000	0.00127	0.01512	0.963	0.556	0.734
64000	0.00112	0.00658	0.916	0.556	0.648
128000	0.00107	0.00403	0.852	0.556	0.534
256000	0.00105	0.00256	0.772	0.556	0.390
512000	0.000952	0.00181	0.720	0.556	0.296
1024000	0.000933	0.00140	0.656	0.556	0.176

Table 25: Simulations in the $M/M/1000+M$ model with $\theta = \mu = 1$ and $\rho = 1.8$, for various lengths of the simulation run.

Run length	ASE(LES)	ASE(EA)	r	$1/\rho$	Error
2000	0.000042	0.000094	0.926	0.714	0.296
4000	0.00219	0.0157	0.990	0.714	0.386
8000	0.00227	0.0293	0.985	0.714	0.378
16000	0.00165	0.0277	0.975	0.714	0.365
32000	0.00135	0.0202	0.971	0.714	0.359
64000	0.00120	0.0146	0.958	0.714	0.342
128000	0.00115	0.0062	0.903	0.714	0.265
256000	0.00110	0.00430	0.862	0.714	0.207
512000	0.00119	0.00363	0.831	0.714	0.163
1024000	0.00125	0.00294	0.780	00.714	0.0926

Table 26: Simulations in the $M/M/1000 + M$ model with $\theta = 0.5$, $\mu = 1$ and $\rho = 1.4$, for various lengths of the simulation run.

Run length	ASE(LES)	ASE(EA)	r	$1/\rho$	Error
2000	0.000042	0.000094	0.926	0.714	0.296
4000	0.000323	0.00250	0.939	0.714	0.314
8000	0.000397	0.00158	0.845	0.714	0.183
16000	0.000256	0.000926	0.845	0.714	0.183
32000	0.000259	0.000721	0.809	0.714	0.133
64000	0.000325	0.000756	0.779	0.714	0.0906
128000	0.000306	0.000571	0.722	0.714	0.0115
256000	0.000285	0.000471	0.691	0.714	0.0329
512000	0.000276	0.000526	0.736	0.714	0.0306
1024000	0.000297	0.000533	0.720	0.714	0.00799

Table 27: Simulations in the $M/M/1000 + M$ model with $\theta = 2.0$, $\mu = 1$ and $\rho = 1.4$, for various lengths of the simulation run.