When Should Doctors and Patients Use Shared Decision-Making Under Bounded Rationality?

Recently, clinicians and governments have increased their advocacy for *shared-decision making* (SDM), a process in which doctors and patients jointly decide amongst appropriate treatment options. Even though both benefits and limitations of SDM have been documented, it is often positioned as a universal recommendation. In contrast, in this paper, we use a stylized analytical model to derive clear guidelines on when and how to employ SDM. Relative to an *evidence-based medicine* (EBM) approach that decides based on population averages, we first establish that doctors should always engage in SDM if both doctors and patients are perfectly rational. However, we show that EBM can outperform SDM once we account for patients' and doctors' bounded rationality (i.e., random errors). We find that when doctors and patients are boundedly rational, administrators should allow doctors to decide whether or not to engage in SDM (versus EBM) on a Case-by-Case (CbC) basis *as long as* doctors are sophisticated enough to make appropriate adjustments to account for such bounded rationality. If doctors are too overconfident (insufficiently accounting for random errors), it can be best to enforce EBM. If doctors are too underconfident (excessively accounting for random errors), it can be best to enforce SDM. More generally, we provide a set of results that map how patient population and doctor characteristics affect the relative performances of SDM, EBM, or CbC decision-making processes.

1. Introduction

Healthcare systems are trending towards patient-centered care (Breen et al. 2010). A cornerstone of patient-centered care is *shared decision-making* (SDM), "a process in which clinicians and patients work together to select tests, treatments, management or support packages, based on clinical evidence and the patient's informed preferences" (Coulter and Collins 2011). SDM promises to improve patient utility, in part, through personalized care. Doctors inform patients about their personalized prognosis regarding risk and outcome predictions, and patients can incorporate their personalized lifestyle and risk preferences. Thus, this "co-production" process promises to capitalize on the heterogeneous needs and preferences of each patient (Daack-Hirsch and Campbell 2014, Bagshaw et al. 2021).

In contrast to SDM, *evidence-based medicine* (EBM) "focuses on using randomized clinical trials (RCTs) to establish the best treatment for the average patient" (de Leon 2012, p. 153). Thus, EBM develops protocols and guidelines based on population averages, rather than focusing on the individual patient (Romana 2006). Advocates for SDM criticize EBM's neglect of the individual patient, calling EBM a "doctor-centered" approach (Sweeney et al. 1998) that overemphasizes the disease and neglects the subjective needs and desires of the patient (Haines et al. 2019, Spatz et al. 2017, JM 2018). These types of criticisms have led to actions such as the United Kingdom's National Health Service (NHS)'s stating that SDM is "appropriate in almost every situation in community, primary and secondary care where a care decision has to be made and that decision is said to be preference-sensitive" (NHS England and NHS Improvement 2019), and marketing the motto "no decision about me without me" (Coulter and Collins 2011). The United States' Patient Protection and Affordable Care Act makes similar recommendations (ACA 2010). And, in 2011, 58 experts from 18 countries published the Salzburg Statement on Shared Decision Making, calling for clinicians and patients to use SDM (Salzburg Global Seminar 2011).

Despite many experts advocating for SDM in general, doctors cite several implementation challenges and are often reluctant or slow to implement it in practice. Patients' lack of information literacy is one such challenge. For example, one medical oncology doctor reports "sometimes I feel like if we lay all the options out there sometimes it confuses them and they are not really making a good decision in the end" (Zeuner et al. 2015). Another says, "the main thing that stands in my way (of using SDM) is the patient's inability to understand risk." (Schoenfeld et al. 2019). Furthermore, doctors often report confusion or uncertainty about when or if they should apply SDM (Baghus et al. 2022, van der Horst et al. 2022, Barker et al. 2019). The frequency with which SDM is used also varies considerably among doctors. Within the same hospital network, one doctor may say, "This is my routine—no decision is made without the involvement of the patient. Clinical practice is about patient education and shared decision making," while another says, "I base my decisions on evidence. I have a manual that I rely on" (Alameddine et al. 2020). Moreover, doctor-specific factors such as experience, confidence, and awareness of one's own limitations often correlate with SDM implementation rates (Simmons et al. 2016, Schoenfeld et al. 2018, 2019, Waddell et al. 2021).

In this paper, we develop clear guidelines for how doctors should approach using SDM by means of a modelling framework that explicitly incorporates *both* the potential benefits of SDM claimed by its advocates *and* the real-world implementation challenges that make doctors reluctant to employ it in practice. That is, we provide answers to the questions: When should healthcare policymakers and administrators mandate that doctors follow SDM or EBM? When should they give doctors the autonomy to decide to use SDM on a *Case-by-Case* (CbC) basis? How do doctor and patient cognitive limitations and behavioral tendencies impact such guidelines?

Doctor-Patient Decision Process	**Description**
Evidence Based Medicine (EBM)	Doctor chooses treatment based on population mean prognoses and preferences
Shared Decision Making (SDM)	Doctor communicates patient-specific prognoses and patient chooses treatment incorporating their personal preferences
Case-by-Case (CbC)	Doctor observes patient-specific prognoses and decides whether or not to employ EBM or SDM

Table 1 Comparison of Doctor-Patient Decision Processes

To gain insight into these questions, we formulate a doctor-patient healthcare decision process with two possible treatments. We develop a stylized model in which patients are heterogeneous in both their medical prognoses under each treatment and their preferences for these medical prognoses. Doctors have private information about patients' medical prognoses under each treatment, and patients have private information about their preferences about medical prognoses. We use our model to compare the performances of the three types of decision-making processes described in Table 1: *Evidence Based Medicine* (EBM), *Shared Decision Making* (SDM), and *Case-by-Case* (CbC).

As shown in Figure 1, we compare those three types of processes under different rationality paradigms which progressively capture more behavioral realism. Under *perfect rationality*, doctors make perfect prognoses and patients make perfect decisions. Under *bounded rationality*, doctors have noisy signals of patient prognoses, and patients sometimes make treatment decision errors when they try to apply their preferences to the prognoses that they are given. When doctors are *sophisticated*, they have the metacognitive ability to properly hedge against such random errors (by making appropriate Bayesian adjustments to their own noisy prognoses signals and properly anticipating patient decision errors). When doctors are *miscalibrated*, they either insufficiently account for random errors (a form of *overconfidence*) or excessively account for random errors (a form of *underconfidence*).

We now outline the structure of the paper and preview our main results. After reviewing the related literature in §2 and introducing the model in §3, we present the following results.

First, in §4, we compare the three types of decision-making processes under a perfectly-rational paradigm. We find that SDM is *always* the right strategy under perfect rationality. Moreover, doctors under CbC always correctly choose whether to employ SDM, making SDM and CbC equivalent.

Second, in §5, we introduce boundedly rational doctors and patients under the assumption that doctors are sophisticated enough to optimally account for such bounded rationality. Under these assumptions, we show that SDM no longer dominates EBM. In fact, it is always optimal to let doctors employ CbC. In other words, bounded rationality can make EBM superior to SDM, and we should let

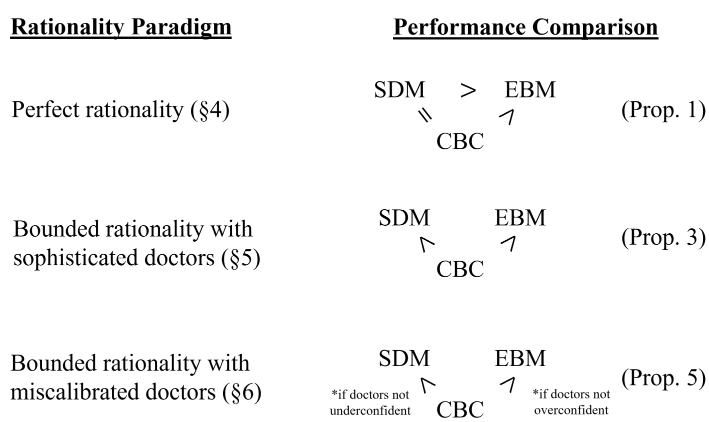


Figure 1 Rationality paradigms and section organization preview

doctors decide when to employ SDM *as long as* they are sophisticated enough to accurately account for such random errors.

Third, in §6, we consider boundedly rational doctors and patients, but relax the assumption of doctor sophistication. Instead, we allow doctors to be miscalibrated by either under- or over-accounting for random errors. Under such conditions, we show that there is no single dominating decision process type. If doctors are overconfident, then it could be beneficial to enforce EBM (CbC dominates SDM). If doctors are underconfident, then it could be beneficial to enforce SDM (CbC dominates EBM).

Finally, in §7, we conduct sensitivity analyses to map how doctor and patient characteristics affect the relative performances of SDM, EBM, and CbC. Table 2 in §7 highlights our main findings. Generally speaking, our analyses suggest that policymakers should:

- Recommend CbC when (i) doctors are accurate and sophisticated, and (ii) patients make large random errors and are medically heterogeneous but have homogeneous preferences.
- Recommend EBM when (i) doctors are inaccurate and overconfident, and (ii) patients make moderate random errors and are medically homogeneous but have heterogeneous preferences.
- Recommend SDM when (i) doctors are inaccurate and underconfident; and (ii) patients make moderate random errors and are medically homogeneous but have heterogeneous preferences.

We conclude, in §8, with a discussion of managerial insights, limitations, and potential future research directions.

2. Related Literature

Shared Decision-Making in the Medical Literature: In general, the medical literature supports the use of SDM in preference-sensitive conditions because it allows for the provision of personalized medicine and the incorporation of patient values into care decisions (Braddock III et al. 1999, Oshima Lee and Emanuel 2013, Veroff et al. 2013, Daack-Hirsch and Campbell 2014, Shay and Lafata 2015). However, some cite patients' lack of health literacy as a major barrier to SDM and caution doctors against engaging patients with low health literacy in decision making (McCaffery et al. 2010, Shippee et al. 2015, Palumbo and Manna 2018). Unfortunately, there are no clear guidelines on when to use SDM (van Veenendaal et al. 2018, Barker et al. 2019, Baghus et al. 2022, van der Horst et al. 2022), leaving such decisions highly dependent on clinician judgment. We contribute to this stream of literature by providing guidelines on when and how to use SDM, taking into account not only the bounded rationality of patients, but also the bounded rationality of doctors.

Shared Decision-Making in the Healthcare Operations Management Literature: The incorporation of patient preferences and patient participation in care decisions has been modeled in the healthcare operations management literature. For example, Ahn and Hornberger (1996) consider incorporating patient preferences for health states in the allocation process for cadaveric kidney transplants, and Batun et al. (2018) consider incorporating patient preferences for risk in liver acceptance decisions for patients with end-stage liver disease. Ayvaci et al. (2018) develop a modeling framework that incorporates patient risk preferences in diagnostic decisions following mammography screening. In optimizing decisions about whether and when to perform biopsies for patients on active surveillance for prostate cancer, Li et al. (2023) allow the weights attributed to the reward criteria to vary according to patient preferences. All of these papers assume that the decision-makers are perfectly rational, whereas we allow for the doctors and patients to be boundedly rational.

Co-production in the Service Operations Management Literature: Co-production in service systems refers to customers playing an active role in the creation of the final output. Co-production has attracted considerable attention in the service operations management literature (Fuchs et al. 1968, Sampson and Froehle 2006), where many papers have analyzed its implications using analytical models. For example, Xue and Field (2008) consider a co-production process with information stickiness in a consulting service and study the work allocation between the consultant and the client as well as the pricing decisions. Roels (2014) identifies the optimal design of a co-production process between a customer and a service provider by investigating how much interaction is needed. The paper finds that as a task becomes less standardized, it is optimal to increase the interaction between the customer and the service provider. Daw et al. (2020) develop new stochastic models for service co-production in contact centers by incorporating dynamic factors that depend on the mechanics of the interaction, such as the number of words written by each party. Some papers have examined the optimal contract design for service co-production. For example, Rahmani et al. (2017) consider a knowledge-intensive project that requires the involvement of both the client and the vendor. They provide several insights into the optimal contract design when the client cannot monitor and verify the vendor's efforts.

Bounded Rationality in Behavioral Operations Management and Judgment and Decision Making: Several papers in behavioral operations management consider the role of bounded rationality (Simon 1957), and noisy decision-making in particular, in designing operational systems. For example, researchers have examined how bounded rationality impacts the optimal design of supply chain contracts (Ho and Zhang 2008, Kalkanci et al. 2011, Su 2008), queues (Huang et al. 2013, Tong and Feiler 2017), auctions (Davis et al. 2014) and forecasting processes (Kremer et al. 2016, Ibrahim et al. 2021) in the presence of human random error. Similarly, this paper examines the role of random errors of doctors and patients on the design of shared decision making.

A key performance issue when individuals make random errors is whether they have the metacognition to make adjustments to account for these errors (e.g., by making a Bayesian correction). Failure to do so leads to overprecision (Soll 1996, Juslin et al. 2007), a form of overconfidence (Moore and Healy 2008). Ren and Croson (2013) show that overprecision is a driver of the well-studied "pull-to-center" behavior in newsvendor decision-making. Feiler and Tong (2022) find that overprecision can interact with system dynamics to result in overoptimism in forecasting new products. Our paper contributes to this literature by examining the role of overprecision (via failure to account for random errors) on the performance and design of medical shared decision making.

3. Model Setting

A doctor and patient need to decide between two treatments: A and B. The utility of treatment t for patient k is equal to

$$U_{kt} = V_k X_{kt}. \tag{1}$$

Here, X_{kt} captures how each patient may medically respond to treatments differently. In other words, X_{kt} denotes the true medical *prognosis* under a given treatment t along a key dimension. We can think of it as the key differentiating feature of a treatment. For example, it could measure the aggressiveness of a treatment (e.g., forecasted variability in quality-of-life outcome), or a key side effect of a treatment (e.g., forecasted weight change). V_k captures how two patients may not like the same medical prognoses equally. That is, V_k denotes patient k's *value* for the attribute of interest, which can be negative.

We also make the following distributional assumptions, primarily to facilitate analysis. We assume that X_{kt} follows a normal distribution with mean μ_t and variance σ^2_{Xt} (we drop the index k because it does not affect the distribution). For algebraic convenience, we assume that $\sigma^2_{XA} = \sigma^2_{XB} = \sigma^2_X/2$, but our results hold even if these variances are not equal. We assume that V_k has a uniform distribution within the domain $[\underline{v}, \bar{v}]$ with mean $\mu_V = \mathbb{E}[V_k]$ and variance $\sigma^2_V = Var[V_k]$, and $\underline{v} < 0 < \bar{v}$. Without loss of generality, we assume that treatment B is "better" in the following average sense: (a) $\mu_B > \mu_A$, and (b) $\mu_V > 0$. In other words, treatment B produces a higher X_{kt} for the average patient, and the average patient prefers positive X_{kt}'s. We assume that V_k's and X_{kt}'s are independent random variables.

In general, doctors are the relative experts in making the medical prognoses for a given patient, while the patient is the relative expert about their own preferences (Ng and Lee 2021, p.4). Therefore, we assume that the patient knows the realization of V_k, but the doctor does not. Similarly, we assume that the doctor observes the realizations of X_{kt} values for a given patient k, but the patient does not. The following two examples may help conceptualize this utility model:

Example 1 *Consider two treatment options for lung cancer: radiation (Treatment A) or surgical extirpation (Treatment B). The latter tends to offer a higher probability of 5-year survival but carries a higher risk of immediate death. Then, we can interpret* X_{kA} *as the level of aggressiveness under treatment A and* X_{kB} *under treatment B, with* $\mu_A - \mu_B < 0$ *because, for most patients, treatment B is more aggressive on average. We can interpret* V_k *as patient k's preference for aggressiveness, with* $\mu_v > 0$ *interpreted as the population of patients generally preferring a more aggressive approach (with higher probability of 5-year survival).*

Example 2 *Consider two treatment options for type-2 diabetes: insulin therapy (Treatment A) and liraglutide therapy (Treatment B). Assume that the focal attribute is the weight loss side effect. Treatment A generally leads to weight gain, while treatment B leads to weight loss (Purnell et al. 2014).*

Then, we can interpret X_{kA} as the weight loss under treatment A and X_{kB} under treatment B, with $\mu_A - \mu_B < 0$ because treatment B leads to greater weight loss on average. We can interpret V_k as patient k's preference to lose weight, with $\mu_V > 0$ interpreted as the average patient preferring to lose weight.

We consider the following three decision-making policies in this paper:

- **Evidence-Based Medicine (EBM):** The doctor selects a treatment based on average patient prognoses and average patient preferences. That is, they solve:

$$EBM: \quad \max_{t \in \{A,B\}} \mathbb{E}[V_k]\mathbb{E}[X_{kt}]. \tag{2}$$

  Given that we have $\mathbb{E}[U_{kA}] - \mathbb{E}[U_{kB}] = \mu_V(\mu_A - \mu_B) < 0$, following EBM in our model set-up is equivalent to always choosing treatment B.

- **Shared-Decision Making (SDM):** For every patient k, the doctor and patient participate in a joint decision-making process: (i) the doctor makes a prediction about the prognosis under each treatment $\hat{X}_{kt} = \hat{x}_{kt}$, and they share these $\hat{x}_{kt}$ values with the patient; (ii) the patient merges $\hat{x}_{kt}$'s provided by the doctor with their preferences captured by $V_k = v_k$ to compare the utilities under each treatment and chooses by solving the problem

$$SDM: \quad \max_{t \in \{A,B\}} \hat{U}_{kt}(v_k, \hat{x}_{kt}), \tag{3}$$

  where $\hat{U}_{kt}(v_k, \hat{x}_{kt})$ denotes the patient's perceived utility for treatment t.

- **Case-by-Case (CbC):** The doctor decides whether to implement EBM or SDM on a patient-by-patient basis: (i) The doctor develops a prediction about the prognosis under each treatment, $\hat{X}_{kt} = \hat{x}_{kt}$; (ii) the doctor decides whether to implement EBM or SDM based on their perceived expected utilities under each process, $U^{SDM}(\hat{x}_{kt})$ and $U^{EBM}(\hat{x}_{kt})$, respectively.[1] Thus, they solve the following problem:

$$\text{CbC:} \quad \max\left\{U^{SDM}(\hat{x}_{kt}), U^{EBM}(\hat{x}_{kt})\right\}. \tag{4}$$

  Under CbC, the probability the doctor implements SDM for patient k is:

$$P_{SDM} = \mathbb{P}_{\hat{X}_{kt}}\left(U^{SDM}(\hat{X}_{kt}) - U^{EBM}(\hat{X}_{kt}) \geq 0\right), \tag{5}$$

  where $\mathbb{P}_{\hat{X}_t}$ denotes the total probability law taken with respect to $\hat{X}_t$.

[1] $U^{SDM}(\hat{x}_{kt})$ and $U^{EBM}(\hat{x}_{kt})$ will be introduced precisely in Sections 4, 5 and 6.

4. Perfect Rationality Benchmark

This section assumes perfectly rational doctors and patients, i.e., doctors make perfectly accurate diagnoses and patients make no errors in combining those diagnoses with their own preferences to make a treatment decision. Thus, for a given patient k, the doctor observes the realization of X_{kt}'s without any error ($\hat{X}_{kt} = X_{kt}$). Then, given $X_{kt} = x_{kt}$ and the patient's realized value $V_k = v_k$, the patient's perceived utility for treatment t, $\hat{U}(v_k, x_{kt})$, is calculated as $\hat{U}_{kt}(v_k, x_{kt}) = v_k x_{kt}$. In this case, the perceived utility is the actual utility for the treatment.

From the doctor's point of view, under SDM, the patient selects treatment A with the following probability:

$$P(V_k, x_{kt}) = \begin{cases} 1, & \text{if } \hat{U}_{kA}(V_k, x_{kA}) \geq \hat{U}_{kB}(V_k, x_{kB}), \\ 0, & \text{otherwise.} \end{cases} \tag{6}$$

Note in (6) that X_{kt} is not random since the doctor observes its realization during the examination, while V_k is a random variable since the doctor does not know its value. Thus, $P(V_k, x_{kt})$ is itself a random variable which is a function of V_k. Furthermore, given $X_{kt} = x_{kt}$, the expected SDM and EBM utilities perceived by a perfectly rational doctor are equal to:

$$\begin{aligned} U^{SDM}(x_{kt}) &= \mathbb{E}\left[V_k X_{kA} P(V_k, X_{kt}) + V_k X_{kB}(1 - P(V_k, X_{kt})) \mid X_{kt} = x_{kt}\right] \\ &= \mathbb{E}_{V_k}\left[V_k x_{kA} P(V_k, x_{kt}) + V_k x_{kB}(1 - P(V_k, x_{kt}))\right], \end{aligned} \tag{7}$$

$$U^{EBM}(x_{kt}) = \mathbb{E}_{V_k}\left[V_k X_{kB} \mid X_{kt} = x_{kt}\right] = \mu_V x_{kB}, \tag{8}$$

where $\mathbb{E}_{V_k}[\cdot]$ denotes the expectation taken with respect to the probability distribution of V_k. The following proposition compares the performances of CbC, SDM, and EBM.

Proposition 1 *Let doctors and patients be perfectly rational. Then,*

(a) Under CbC, the doctor selects SDM with probability 1.

(b) SDM (or, equivalently, CbC) provides a higher expected utility than EBM.

Proposition 1 establishes that SDM outperforms EBM if doctors and patients are perfectly rational. Moreover, CbC is equivalent to SDM because under CbC the doctor will always choose to implement SDM. The intuition is that SDM optimally uses a personalized medical diagnosis, X_{kt}, and the personal preferences of the patient, V_k, to make a treatment decision. In contrast, EBM is based only on population averages. CbC always chooses SDM because the doctor recognizes that such personalization achieved by SDM outperforms EBM in expectation.

5. Bounded Rationality with Sophisticated Doctors

In Section 4, we considered perfectly rational doctors and patients. In reality, doctors are humans whose judgments about a patient's medical prognosis may suffer from random error (Gigerenzer and Muir Gray 2011, Kahneman et al. 2016). Furthermore, patients may make errors in their attempts to interpret the information provided by the doctor and to combine it with their personal preferences to make a decision, for example, due to patient illiteracy and innumeracy (Williams et al. 2002). Therefore, in this section, we relax the assumption of perfect rationality for doctors and patients by incorporating random error. We introduce two new parameters $\sigma_d \geq 0, \sigma_p \geq 0$ which capture the variance of doctors' and patients' random errors, respectively. When $\sigma_d = \sigma_p = 0$, the model reduces to the perfectly rational model in Section 4.

Though doctors and patients are boundedly rational in the sense that they suffer from random error, we first assume that doctors are *sophisticated* in that they recognize and account for such random errors intelligently. Specifically, we assume they make Bayesian corrections to account for the noise in their medical diagnoses. And, they take expectation over the random error of their patients when deciding whether or not to engage in SDM. In Section 6, we relax this sophistication assumption.

We begin by describing our model for the doctor. We assume that the doctor does not know the actual realizations of the prognoses under treatments A and B, i.e., X_{kA} and X_{kB}. Rather, through their examination of a given patient k, they can only observe a noisy signal of the prognosis, S_{kt}:

$$S_{kt} := X_{kt} + \mathcal{E}_{kt}, \tag{9}$$

where $\mathcal{E}_{kt}$'s are identically normally distributed random variables with mean zero and variance $\sigma_d^2/2$. Here, σ_d captures the doctor's inconsistencies or random errors in evaluating the patient's condition through the examination. We assume that $\mathcal{E}_{kt}$'s are independent across the treatments and across patients, and are also independent of V_k's and X_{kt}'s. Given the noisy signal, S_{kt}, the doctor's prediction for each treatment's prognosis, $\hat{X}_{kt}$, is equal to the Bayesian-adjusted forecast:

$$\hat{X}_{kt} = \mathbb{E}[X_{kt} \mid S_{kt}] = \frac{\sigma_X^2}{\sigma_X^2 + \sigma_d^2} S_{kt} + \frac{\sigma_d^2}{\sigma_X^2 + \sigma_d^2} \mathbb{E}[X_{kt}]. \tag{10}$$

Thus, the doctor's forecast for the difference between the two treatments' prognoses is:

$$\hat{X}_{kA} - \hat{X}_{kB} = \frac{\sigma_X^2}{\sigma_X^2 + \sigma_d^2}(S_{kA} - S_{kB}) + \frac{\sigma_d^2}{\sigma_X^2 + \sigma_d^2}(\mu_A - \mu_B). \tag{11}$$

We now turn to describing our model for the patient. Given the doctor's noisy prognosis $\hat{X}_{kt} = \hat{x}_{kt}$ and the patient's individual value $V_k = v_k$, the patient predicts the utility of treatment t to be:

$$\hat{U}_{kt}(v_k, \hat{x}_{kt}) = v_k \hat{x}_{kt} + \gamma_{kt}, \tag{12}$$

where γ_{kt} represents the random error added by the patient. We assume that γ_{kt}'s are identically distributed random variables which are independent across the treatments, across patients, and from

V_k's, X_{kt}'s, and $\mathcal{E}_{kt}$'s. We assume that γ_{kt} follows an extreme value distribution with standard deviation $\frac{\pi}{\sqrt{6}}\sigma_p$. Here, σ_p captures the patient's irrationality level. The extreme value distribution assumption for the error terms is common in the bounded rationality literature (Anderson et al. 1992). To that end, from the doctor's point of view, under SDM, the patient selects treatment A with the following probability, given $\hat{X}_{kt} = \hat{x}_{kt}$, where $\mathbb{P}_\gamma$ denotes the total probability law taken with respect to γ:

$$P(V_k, \hat{x}_{kt}) = \mathbb{P}_\gamma\left(\hat{U}_{kA}(V_k, \hat{x}_{kA}) \geq \hat{U}_{kB}(V_k, \hat{x}_{kB})\right) = \frac{1}{1 + \mathrm{e}^{V_k(\hat{x}_{kB} - \hat{x}_{kA})/\sigma_p}}. \tag{13}$$

As in Section 4, the selection probability of treatment A, $P(V_k, \hat{x}_{kt})$, is a random variable from the doctor's perspective, as it is a function of V_k which is not observed by the doctor. Furthermore, given $\hat{X}_{kt} = \hat{x}_{kt}$, the expected SDM and EBM utilities perceived by the doctor are:

$$\begin{aligned} U^{SDM}(\hat{x}_{kt}) &= \mathbb{E}\left[V_k \hat{X}_{kA} P(V_k, \hat{X}_{kt}) + V_k \hat{X}_{kB}(1 - P(V_k, \hat{X}_{kt})) \mid \hat{X}_{kt} = \hat{x}_{kt}\right] \\ &= \mathbb{E}_{V_k}\left[V_k \hat{x}_{kA} P(V_k, \hat{x}_{kt}) + V_k \hat{x}_{kB}(1 - P(V_k, \hat{x}_{kt}))\right], \end{aligned} \tag{14}$$

$$U^{EBM}(\hat{x}_{kt}) = \mathbb{E}_{V_k}\left[V_k \hat{X}_{kB} \mid \hat{X}_{kt} = \hat{x}_{kt}\right] = \mu_V \hat{x}_{kB}. \tag{15}$$

However, since doctors have noisy signals of patients' prognoses, the actual expected SDM and EBM utilities given the noisy signal $S_{kt} = s_{kt}$ may differ from the doctor's perceived expected utilities introduced in (14) and (15). Specifically, given that the doctor, after observing the signal $S_{kt} = s_{kt}$, communicates to the patient the forecast $\hat{x}_{kt} = \mathbb{E}[X_{kt} | S_{kt} = s_{kt}]$ (recall (10) and note that $\hat{x}_{kt}$ is a function of s_{kt}), the actual expected SDM and EBM utilities, denoted by $U_a^{SDM}(s_{kt})$ and $U_a^{EBM}(s_{kt})$, respectively, are:

$$\begin{aligned} U_a^{SDM}(s_{kt}) &= \mathbb{E}_{X_{kt}, V_k}\left[V_k X_{kA} P(V_k, \hat{x}_{kt}) + V_k X_{kB}(1 - P(V_k, \hat{x}_{kt}) \mid S_{kt} = s_{kt}\right] \\ &= \mathbb{E}_{V_k}\left[V_k \mathbb{E}[X_{kA} \mid S_{kA} = s_{kA}] P(V_k, \hat{x}_{kt}) + V_k \mathbb{E}[X_{kB} \mid S_{kB} = s_{kB}](1 - P(V_k, \hat{x}_{kt}))\right], \end{aligned} \tag{16}$$

$$U_a^{EBM}(s_{kt}) = \mathbb{E}_{X_{kt}, V_k}\left[V_k X_{kB} \mid S_{kB} = s_{kB}\right] = \mu_V \mathbb{E}[X_{kB} \mid S_{kB} = s_{kB}]. \tag{17}$$

Since the actual expected SDM and EBM utilities may be different from those perceived by the doctor, it is possible that under CbC, the doctor may choose the "wrong" strategy, i.e., they may engage in SDM when they should not have (in an average sense), which we refer to as *wrong SDM*, or they may not engage in SDM when they should have (in an average sense), which we refer to as *wrong EBM*. To this end, under CbC, we define P_W^{SDM}, P_W^{EBM}, and P_W as the probability of wrong SDM, the probability of wrong EBM, and the total probability of following the wrong strategy, respectively. Using (14)-(17):

$$P_W^{SDM} = \mathbb{P}_{S_{kt}}\left(U^{SDM}(\hat{X}_{kt}) - U^{EBM}(\hat{X}_{kt}) \geq 0,\, U_a^{SDM}(S_{kt}) - U_a^{EBM}(S_{kt}) < 0\right), \tag{18}$$

$$P_W^{EBM} = \mathbb{P}_{S_{kt}}\left(U^{SDM}(\hat{X}_{kt}) - U^{EBM}(\hat{X}_{kt}) < 0,\, U_a^{SDM}(S_{kt}) - U_a^{EBM}(S_{kt}) \geq 0\right), \tag{19}$$

$$P_W = P_W^{SDM} + P_W^{EBM}. \tag{20}$$

We emphasize that "right" and "wrong" strategies are to be understood in an ex-ante average sense, e.g., when a doctor selects a wrong strategy (based on comparing the ex-ante expectations), they may actually still select the right treatment ex-post (based on the specific realizations of the random variables for the patient).

5.1. Performance Comparisons

We now compare the performances of CbC, SDM, EBM. To do so, it is helpful to first characterize the doctor's SDM engagement probability and wrong SDM and EBM probabilities under CbC.

Proposition 2 *Let doctors and patients be boundedly rational and doctors sophisticated. Then, under CbC:*

(a) The doctor does not always select SDM, i.e., $P_{SDM} < 1$.

(b) The doctor selects the right strategy for all patients, i.e. $P_W^{SDM} = P_W^{EBM} = P_W = 0$.

Recall from Proposition 1(a) in Section 4 that under perfect rationality, a doctor under CbC always chooses to employ SDM. In contrast, Proposition 2(a) shows that with bounded rationality, a doctor under CbC selects EBM instead of SDM for some patients. Proposition 2(b) states that doctors under CbC optimally choose between SDM and EBM for all patients. In other words, sophisticated doctors do the best they can given the information they have. Of course, because their prognoses are not perfect and patient behavior is noisy, these choices may not be optimal ex post.

Proposition 2 guarantees that under CbC, the doctor always chooses the strategy that provides a higher actual conditional expected utility given the signal. To compare the performances of CbC, SDM, and EBM, on the other hand, we define the expected utilities of these three policies with unconditional expectations calculated by averaging the actual conditional expected utilities over erroneous signals, thereby accounting for the errors in the signals themselves. Recall from Proposition 1 in Section 4 that SDM provides the same utility as CbC, and both SDM and CbC provide a higher utility than EBM if patients and doctors are perfectly rational. However, in this section, doctor and patient errors pose a disadvantage for both SDM and CbC, and deteriorate their performances. Hence, it is not obvious whether SDM and CbC still dominate EBM. Furthermore, given that human random error breaks the equivalence between CbC and SDM, as per Proposition 2(a), it is unclear, a priori, whether CbC will perform better than SDM. Nevertheless, the following proposition establishes that with sophisticated doctors, CbC does in fact always generate a higher expected utility than SDM.

Proposition 3 *Let doctors and patients be boundedly rational and doctors sophisticated. Then:*

(a) SDM provides a lower expected utility than EBM under certain conditions, e.g., when σ_p *is sufficiently high.*

(b) CbC provides a higher expected utility than both EBM and SDM.

Proposition 3(a) shows that with human random error, it is not possible to ensure the superiority of SDM over EBM for all conditions. Indeed, it can be shown that when both doctors and patients are erroneous, using SDM rather than EBM results in a utility loss for sufficiently low patient rationality (high σ_p), underscoring the importance for policymakers to recognize patient and doctor errors when adopting appropriate policies. This result contrasts with the conventional wisdom of always using SDM (Couët et al. 2015, Stiggelbout et al. 2015, Merchant et al. 2018, NHS England and NHS Improvement 2019).

Furthermore, Proposition 3(b) demonstrates that if doctors are sophisticated, then the expected utility generated by CbC is higher than the ones generated by pure EBM or pure SDM. The rationale is as follows. On the one hand, unlike EBM, the doctor under CbC can involve a patient in the decision-making process if they predict that the patient will probably select the "correct" treatment, matching the patient's values. For such patients, CbC enables benefiting from SDM's patient-value incorporation and personalized medicine. On the other hand, once the doctor chooses to engage in SDM, they subject themselves to both their own forecasting errors and patient errors. As such, to ensure that CbC outperforms both SDM and EBM, it becomes crucial for doctors in CbC to be able to switch between EBM and SDM correctly. Sophisticated doctors are able to do so under CbC, as per Proposition 2(b). Hence, CbC incorporates a patient into SDM only if that patient is likely to select the treatment that reflects their individual preferences.

6. Bounded Rationality with Miscalibrated Doctors

Doctors may not be sophisticated enough to optimally account for random errors, as we assumed in §5. In particular, in this section we consider doctors who may be *miscalibrated* in their attempt to account for random errors, either by insufficiently or excessively accounting for them. We introduce two new behavioral parameters $\alpha_d \geq 0$ and $\alpha_p \geq 0$ that can capture doctors who are miscalibrated with respect to their own prognosis' random error and patient random errors, respectively. When $\alpha_d = \alpha_p = 1$, the doctor is sophisticated and the model reduces to that in §5.

When $\alpha_d = 1$, the doctor is Bayesian. When $\alpha_d < 1$, the doctor insufficiently accounts for their prognosis random error. The doctor over-relies on the signal and insufficiently accounts for prior, consistent with base-rate neglect (Kahneman and Tversky 1973, Bar-Hillel 1980). When $\alpha_d > 1$, the doctor excessively accounts for their prognosis random error. The doctor does not respond to the signal sufficiently and over-relies on the prior, consistent with conservatism (Edwards 1968). In general, we refer to insufficiently accounting for random error as a form of *overconfidence*, and excessive accounting for random error as *underconfidence*.

The doctor's prognosis for patient k under treatment t is:

$$\hat{X}^m_{kt} = \frac{\sigma_X^2}{\sigma_X^2 + \alpha_d \sigma_d^2} S_{kt} + \frac{\alpha_d \sigma_d^2}{\sigma_X^2 + \alpha_d \sigma_d^2} \mathbb{E}[X_{kt}], \tag{21}$$

The doctor's belief about the difference between prognoses under the two treatments is:

$$\hat{X}_{kA}^m - \hat{X}_{kB}^m = \frac{\sigma_X^2}{\sigma_X^2 + \alpha_d \sigma_d^2}(S_{kA} - S_{kB}) + \frac{\alpha_d \sigma_d^2}{\sigma_X^2 + \alpha_d \sigma_d^2}(\mu_A - \mu_B). \tag{22}$$

Next, for patient k, the miscalibrated doctor's perceived selection probability for treatment A, under SDM, is equal to:

$$P^m(V_k, \hat{x}_{kt}^m) = \frac{1}{1 + \mathrm{e}^{V_k(\hat{x}_{kB}^m - \hat{x}_{kA}^m)/(\alpha_p \sigma_p)}}. \tag{23}$$

We note that we write (23) by replacing σ_p in (13) with $\alpha_p \sigma_p$, and α_p reflects the degree to which the miscalibrated doctor accounts for patient errors. When $\alpha_p = 1$, the doctor is sophisticated and accurately accounts for patient random errors. When $\alpha_p < 1$, the doctor insufficiently accounts for patient decision random error, and when $\alpha_p > 1$ the doctor excessively accounts for it.

For miscalibrated doctors, the doctor's perceived expected utilities from implementing SDM or EBM may differ from the true expected utilities. We define the doctor's belief about the expected SDM and EBM utilities, $U^{SDM}(\hat{x}_{kt}^m)$ and $U^{EBM}(\hat{x}_{kt}^m)$, as follows:

$$\begin{aligned} U^{SDM}(\hat{x}_{kt}^m) &= \mathbb{E}\left[V_k \hat{X}_{kA}^m P^m(V_k, \hat{X}_{kt}^m) + V_k \hat{X}_{kB}^m (1 - P^m(V_k, \hat{X}_{kt}^m)) \mid \hat{X}_{kt}^m = \hat{x}_{kt}^m\right] \\ &= \mathbb{E}_{V_k}\left[V_k \hat{x}_{kA}^m P^m(V_k, \hat{x}_{kt}^m) + V_k \hat{x}_{kB}^m (1 - P^m(V_k, \hat{x}_{kt}^m))\right], \end{aligned} \tag{24}$$

$$U^{EBM}(\hat{x}_{kt}) = \mathbb{E}_{V_k}\left[V_k \hat{X}_{kB}^m \mid \hat{X}_{kt}^m = \hat{x}_{kt}^m\right] = \mu_V \hat{x}_{kB}^m. \tag{25}$$

On the other hand, the actual expected utilities of SDM and EBM for a doctor who shares the forecast $\hat{X}_{kt}^m = \hat{x}_{kt}^m$ are:

$$\begin{aligned} U_a^{SDM}(s_{kt}) &= \mathbb{E}_{X_{kt}, V_k}\left[V_k X_{kA} P(V_k, \hat{x}_{kt}^m) + V_k X_{kB}(1 - P(V_k, \hat{x}_{kt}^m) \mid S_{kt} = s_{kt}\right] \\ &= \mathbb{E}_{V_k}\left[V_k \mathbb{E}[X_{kA} \mid S_{kA} = s_{kA}] P(V_k, \hat{x}_{kt}^m) + V_k \mathbb{E}[X_{kB} \mid S_{kB} = s_{kB}](1 - P(V_k, \hat{x}_{kt}^m))\right], \end{aligned} \tag{26}$$

$$U_a^{EBM}(s_{kt}) = \mathbb{E}_{X_{kt}, V_k}\left[V_k X_{kB} \mid S_{kB} = s_{kB}\right] = \mu_V \mathbb{E}[X_{kB} \mid S_{kB} = s_{kB}]. \tag{27}$$

Note that in (24)-(27), we use the independence of S_{kt} and V_k to arrange the expressions for the expectations.

6.1. Performance Comparisons

To compare the performances of SDM, EBM, and CbC, it is again helpful to first characterize the doctor's wrong SDM and EBM probabilities under CbC.

Proposition 4 *Let doctors and patients be boundedly rational and doctors be miscalibrated. Then, under CbC, the following statements hold:*

(a) If the doctor is overconfident ($\alpha_d < 1, \alpha_p < 1$)*, then* $P_W^{SDM} > 0$ *and* $P_W^{EBM} = 0$*.*

(b) If the doctor be underconfident ($\alpha_d > 1, \alpha_p > 1$)*, then there exists a constant* Φ *such that:*

(i) If $\alpha_p\sigma_p \geq \Phi$, then $P_W^{SDM} = 0$ and $P_W^{EBM} > 0$.

(ii) If $\alpha_p\sigma_p < \Phi$, then $P_W^{SDM} > 0$ and $P_W^{EBM} > 0$. Furthermore, $P_{SDM} - P_W^{SDM} > P_W^{SDM}$.

(see Appendix B for the exact expression of Φ).

Recall from Proposition 2(b) in Section 5 that sophisticated doctors make optimal SDM/EBM decisions for all patients. In contrast, Proposition 4 shows that, with miscalibrated doctors, the doctor engages in wrong SDM or wrong EBM for some patients. Specifically, Proposition 4(a) establishes that overconfident doctors tend to overestimate the expected utility difference between SDM and EBM by not sufficiently recognizing the prognosis random errors and patient random errors. This creates a systematic bias in the doctor's SDM engagement decisions, resulting in engaging in wrong SDM for some patients, i.e., $P_W^{SDM} > 0$. On the other hand, Proposition 4(b) demonstrates that underconfident doctors tend to engage in wrong EBM. That is, they either never conduct wrong SDM and only conduct wrong EBM for some patients (when the perceived patient noise $\alpha_p\sigma_p$ is sufficiently high; see part (i) of Proposition 4(b)), or they may conduct both wrong SDM for some patients and wrong EBM for others, with a wrong SDM probability that is lower than the right SDM probability (when $\alpha_p\sigma_p$ is sufficiently low; see part (ii) of Proposition 4(b)).

We next compare the performances of CbC, SDM, and EBM. First, recall from Section 5 that with bounded rationality and sophisticated doctors, SDM may sometimes provide a lower utility than EBM. Similarly, here, with bounded rationality and miscalibrated doctors, it is still possible that EBM outperforms SDM.

Second, Proposition 3(b) in Section 5 establishes that with sophisticated doctors, who optimally decide between SDM and EBM, CbC generates a higher expected utility than both pure SDM and pure EBM. In contrast, in this section, miscalibrated doctors fail to make optimal SDM/EBM decisions under CbC as per Proposition 4, which deteriorates the performance of CbC. Hence, it is not apparent, a priori, whether CbC will continue to perform better than both SDM and EBM. Nevertheless, the following proposition demonstrates that with miscalibrated doctors, CbC, in fact, yields a higher expected utility than SDM so long as the doctor is not underconfident, whereas it yields a higher expected utility than EBM so long as the doctor is not overconfident.

Proposition 5 *Let doctors and patients be boundedly rational and doctors miscalibrated. Then, the following statements hold:*

(a) If the doctor is not underconfident ($\alpha_d \leq 1, \alpha_p \leq 1$), then CbC provides a higher expected utility than SDM.

(b) If the doctor is not overconfident ($\alpha_d \geq 1, \alpha_p \geq 1$), then CbC provides a higher expected utility than EBM.

Proposition 5(a) shows that, with overconfident doctors, CbC generates a larger expected utility than pure SDM. The reason is as follows. Under SDM, doctors engage in SDM for all patients, whereas under CbC they do not allow a patient to be involved in decisions if they predict that SDM will produce a lower utility than EBM for that patient. This reduces the exposure to both the doctor's and patients' random errors, and improves the performance of CbC against SDM. If the doctor conducts wrong EBM, then the performance of CbC deteriorates against SDM. However, Proposition 4(a) ensures that overconfident doctors never conduct wrong EBM. Since under CbC, underconfident doctors tend to engage in wrong EBM for some patients, as established by Proposition 4(b), it is possible that SDM outperforms CbC with underconfident doctors, e.g., when doctors are sufficiently accurate.

Furthermore, Proposition 5(b) establishes that if doctors are underconfident, CbC generates a larger expected utility than pure EBM. The rationale is as follows. Although CbC, unlike EBM, provides flexibility by enabling doctors to switch to SDM and to benefit from personalized medicine and preference incorporation for some patients, the performance of CbC deteriorates against EBM when the doctor engages in wrong SDM. However, Proposition 4(b) ensures that underconfident doctors are not very likely to conduct wrong SDM, i.e., they conduct right SDM more often than wrong SDM. Since overconfident doctors under CbC, unlike underconfident doctors, suffer from wrong SDM, which may be more frequent than right SDM, it is possible that EBM may outperform CbC with overconfident doctors, e.g., when doctors and patients make large random errors.

The fact that EBM or SDM can sometimes outperform CbC with miscalibrated doctors sheds light on the value of accounting for doctor limitations when developing guidelines for shared decision-making. That is, if one were to account only for patient errors but not for doctors' cognitive biases (i.e., overconfidence and underconfidence), one would erroneously consider CbC to be superior to EBM (see Proposition 3(b)). However, Proposition 5 shows that policymakers should be cautious in recommending miscalibrated doctors employ CbC rather than EBM or SDM. This is an important insight, especially since the medical literature often reports patients' lack of health literacy as a major barrier to SDM, but pays little attention to doctors' limitations (McCaffery et al. 2010, Shippee et al. 2015, Palumbo and Manna 2018).

7. Sensitivity Analysis: When is CbC, SDM, or EBM the right process?

In Sections 4, 5, and 6, we analytically compared the performances of CbC, SDM, and EBM under progressively more general behavioral assumptions. We found that while SDM is always the best process when doctors and patients are perfectly rational, this is no longer the case once you allow for random errors. With random errors, CbC is best if doctors are sophisticated, but SDM or EBM could be best if doctors are miscalibrated. So, in general, there is no single dominating treatment process.

When should policymakers recommend CbC, SDM, or EBM? To answer this question, we conduct sensitivity analysis to compare the expected utilities of CbC, SDM, or EBM, for different doctor and patient characteristics. We first establish some results analytically and then perform numerical analyses. Finally, we provide insights into when CbC, SDM, or EBM are superior.

7.1. Analytical Results

We first characterize the monotonicity of the expected utility difference between SDM and EBM with respect to various system parameters; see Appendix B for the precise formulation of the sufficient conditions listed in Proposition 6.

Proposition 6 *Let doctors and patients be boundedly rational and doctors be miscalibrated. Then, the expected utility difference between SDM and EBM (a) increases in σ_V if σ_V is sufficiently large, (b) increases in σ_X if σ_X is sufficiently large, (c) decreases in σ_p if σ_p is sufficiently large, and (d) decreases in σ_d if σ_d is sufficiently low, $\alpha_d \geq 1/2$, and $\sigma_X \geq -(\mu_A - \mu_B)\sqrt{2\alpha_d - 1}$.*

SDM enables doctors to integrate patients' preferences into treatment decisions and provide personalized information to the patients about the treatment prognoses. Despite such advantages, patients may choose the wrong treatment, i.e., the treatment that does not maximize their utility, due to their own errors and noisy forecasts communicated by the doctor. In contrast, since EBM always prescribes the optimal treatment for the average patient, factors such as patient preference heterogeneity, prognosis variability, patient and doctor errors, as well as doctors' underconfidence or overconfidence degree, do not impact its performance.

Proposition 6(a) and 6(b) establish that the benefit of SDM becomes larger relative to EBM when patients are more heterogeneous in their preferences (high σ_V), or treatment prognoses are more variable across the patient population (high σ_X). Proposition 6(c) and 6(d) show that an increase in the level of patient random errors, σ_p, and an increase in the level of doctor inaccuracy, σ_d (if doctors are not too overconfident, i.e., α_d is not too low, and prognosis variability, σ_X, is sufficiently high) increase the risk of selecting the wrong treatment, thereby worsening the performance of SDM relative to EBM.

Moreover, the key insights of Proposition 6 apply to the performance comparisons between CbC and EBM on one hand, and between CbC and SDM on the other hand. Specifically, if patient value heterogeneity (σ_V), treatment prognosis variability (σ_X), and doctor accuracy (σ_d) are low, and patients make large errors (high σ_p), then we expect that (i) CbC would lead to a lower expected utility than EBM, and (ii) SDM would lead to a lower expected utility than CbC. This is because CbC is more error-sensitive than EBM, and SDM is more error-sensitive than CbC.

7.2. Numerical Study

In this subsection, we conduct a numerical study to identify the settings under which CbC, EBM, or SDM perform the best by far. We first define some new parameters $P_{E,d}$, $P_{E,p}$, P_X and P_V, which can be interpreted as normalizations of σ_p, σ_d, σ_X, and σ_V, respectively:

$$P_{E,d} = \mathbb{P}_{X_{kt},\varepsilon_{kt}}(X_{kA} > X_{kB},\, S_{kA} < S_{kB}) + \mathbb{P}_{X_{kt},\varepsilon_{kt}}(X_{kA} < X_{kB},\, S_{kA} > S_{kB}),$$

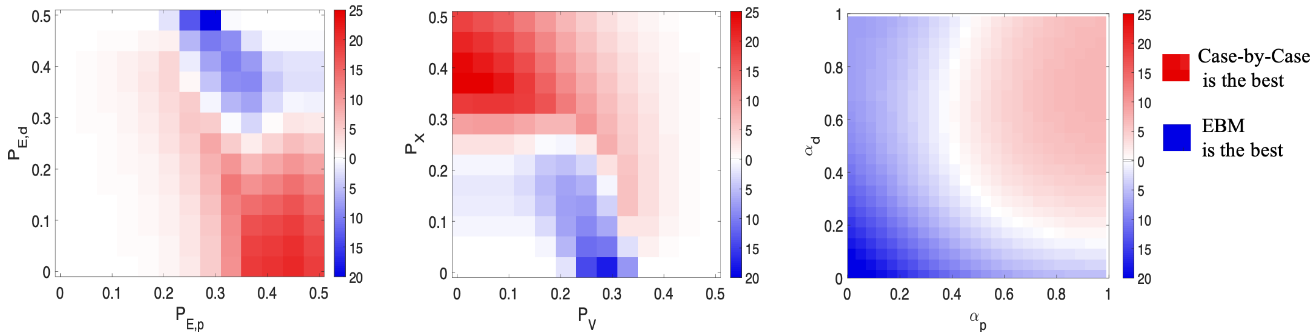
$$
\begin{aligned}
P_{E,p} = \mathbb{P}_{V_k,S_{kt},\gamma_{kt}}(V_k\hat{X}_{kA} > V_k\hat{X}_{kB},\, V_k\hat{X}_{kA}+\gamma_{kA} < V_k\hat{X}_{kB}+\gamma_{kB}) \\
+\mathbb{P}_{V_k,S_{kt},\gamma_{kt}}(V_k\hat{X}_{kA} < V_k\hat{X}_{kB},\, V_k\hat{X}_{kA}+\gamma_{kA} > V_k\hat{X}_{kB}+\gamma_{kB}), \\
P_X = \mathbb{P}_{X_{kt}}(X_{kA} > X_{kB}), \qquad P_V = \mathbb{P}_{V_k}(V_k < 0).
\end{aligned}
$$

We note that $P_{E,d}$, $P_{E,d}$, P_X and P_V increase in σ_d, σ_p, σ_X and σ_V, respectively, and they all lie within the interval $[0, 0.5]$. Indeed, $P_{E,d}$ and $P_{E,p}$ capture the level of doctor noise (σ_d) and patient error (σ_p), respectively, whereas P_X and P_V help us to quantify the level of prognosis variability (σ_X) and patient value heterogeneity (σ_V), respectively. In our base case scenario, we set $P_{E,d}$, $P_{E,p}$, P_X, P_V, α_d and α_p to moderate values, which we believe is consistent with a realistic setting. In particular, for the base case, we let $P_{E,d} = P_{E,p} = 0.30$, $P_X = P_V = 0.25$, $\alpha_d = \alpha_p = 0.50$ in the experiments with overconfident doctors, and we let $P_{E,d} = P_{E,p} = 0.30$, $P_X = P_V = 0.30$, $\alpha_d = \alpha_p = 2$ in the experiments with underconfident doctors. We vary two parameters at a time while keeping others constant.

Figures 2 and 3 illustrate when CbC, EBM, or SDM is the right policy for overconfident and underconfident doctors, respectively, as a function of the pairs $(P_{E,p}, P_{E,d})$, (P_V, P_X), and (α_p, α_d). The darker red in these figures indicates a higher percentage utility improvement through switching to CbC from the second-best policy, the darker blue indicates a higher percentage utility improvement through switching to EBM from the second-best policy, whereas the darker green indicates a higher percentage utility improvement through switching to SDM from the second-best policy. We make three observations.

First, the blue regions in Figure 2 indicate the settings for which EBM yields substantially higher expected utility than CbC (the second-best policy) with an overconfident doctor (recall from Section 6 that SDM is always suboptimal for overconfident doctors). We observe that EBM performs better than CbC (i) with highly inaccurate doctors and moderate patient errors (high σ_d and moderate σ_p; see Figure 2(a)), (ii) with low prognosis heterogeneity and moderate patient value heterogeneity (low σ_X and moderate σ_V; see Figure 2(b)), and (iii) with highly overconfident doctors (low α_d and α_p; see Figure 2(c)). In such settings, performing EBM instead of CbC can increase the expected utility by up to 20%.

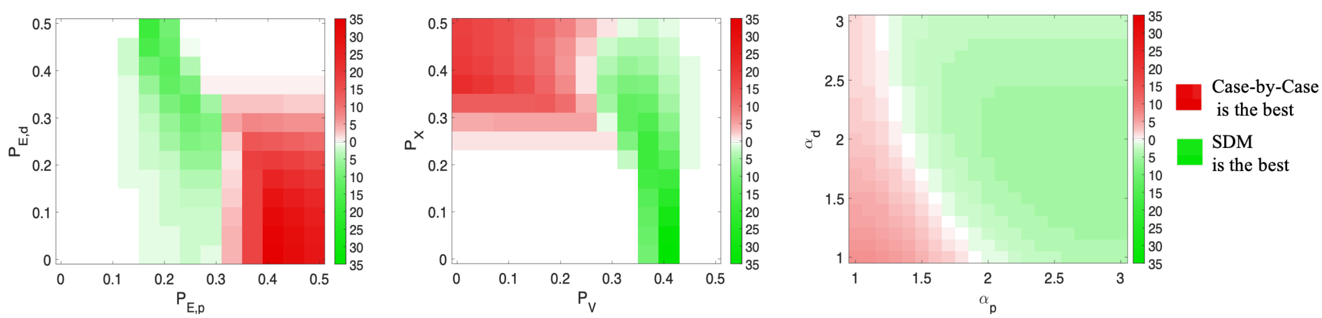
The significantly better performance of EBM over CbC in settings with low σ_X and high σ_d is consistent with Proposition 6. Furthermore, for EBM to outperform CbC, it is expected that doctors are not too sophisticated in accounting for random errors (α_d and α_p should be sufficiently below 1). This is because with sophisticated doctors, CbC is superior to EBM (recall Proposition 3(b)). Moreover, although a sufficiently low level of patient-value heterogeneity (σ_V) and a sufficiently high level of patient errors (σ_p) are enough to ensure the superiority of EBM over CbC, moderate σ_p and σ_V values are needed for EBM to yield significantly higher expected utility than CbC. With a not too large value of prognosis variability (σ_X), the doctor under CbC chooses to conduct EBM most of



(a) The effect of $P_{E,p}$ and $P_{E,d}$ (b) The effect of P_V and P_X (c) The effect of α_p and α_d

Figure 2 Overconfident doctors: The percentage improvement in the expected utilities provided by switching to CbC (red) and EBM (blue) when $\mu_A = -1$, $\mu_B = 1$, $\mu_V = 1$, $P_X = P_V = 0.25$, $P_{E,p} = P_{E,d} = 0.30$, $\alpha_p = \alpha_d = 0.5$.

Note: In the red regions, CbC is the best policy, whereas in the blue regions, EBM is the best policy. Given that EU^{CBC}, EU^{EBM} and EU^{SDM} denote the expected utility under CBC, EBM, and SDM, respectively, the darker red indicates a higher $\frac{EU^{CBC} - \max\{EU^{EBM}, EU^{SDM}\}}{EU^{CBC}} \times 100$ value, whereas the darker blue indicates a higher $\frac{EU^{EBM} - \max\{EU^{CBC}, EU^{SDM}\}}{EU^{EBM}} \times 100$ value.



(a) The effect of $P_{E,p}$ and $P_{E,d}$ (b) The effect of P_V and P_X (c) The effect of α_p and α_d

Figure 3 Underconfident doctors: The percentage improvement in the expected utilities provided by switching to CbC (red) and SDM (green) when $\mu_A = -1$, $\mu_B = 1$, $\mu_V = 1$, $P_X = P_V = P_{E,p} = P_{E,d} = 0.30$, $\alpha_p = \alpha_d = 2$.

Note: In the red regions, CbC is the best policy, whereas in the green regions, SDM is the best policy. Given that EU^{CBC}, EU^{EBM} and EU^{SDM} denote the expected utility under CBC, EBM, and SDM, respectively, the darker red indicates a higher $\frac{EU^{CBC} - \max\{EU^{EBM}, EU^{SDM}\}}{EU^{CBC}} \times 100$ value, whereas the darker green indicates a higher $\frac{EU^{SDM} - \max\{EU^{CBC}, EU^{EBM}\}}{EU^{SDM}} \times 100$ value.

the time, and thus EBM performs close to CbC if patient value heterogeneity (σ_V) is very low or the level of patient errors (σ_p) is very high.

Second, CbC is the best policy when (i) doctors are sufficiently accurate but patients make large errors (low σ_d and high σ_p; see Figures 2(a) and 3(a)), (ii) the prognoses under the two treatments have sufficiently large variability across patients, and patients are not too heterogeneous in their preferences (high σ_X and low σ_V; see Figures 2(b) and 3(b)), and (iii) doctors are sufficiently sophisticated (α_p and α_d values close to 1; see Figures 2(c) and 3(c)). In such settings, switching from SDM or EBM to CbC can increase the expected utility by up to 35%.

The substantial superiority of CbC over SDM and EBM with sophisticated doctors (α_p and α_d values close to 1) is in line with our previous results in Proposition 3(b). Proposition 6 implies that an increase in prognosis variability (σ_X) improves the performance of CbC over EBM, while reducing the expected utility gap between CbC and SDM. Despite these two effects of increasing σ_X, the need for a sufficiently large σ_X for CbC to provide significantly greater utility than EBM and SDM is based on the following intuition. When prognosis variability is very low, CbC reduces to either SDM or EBM, which in turn results in CbC performing very closely to either EBM or SDM. Similarly, the doctor's inaccuracy level (σ_d) should not be too high for the expected utility of CbC to be much greater than that of EBM or SDM, because too inaccurate doctors' forecasts for the treatment prognoses become very close to the mean prognosis values, $\mathbb{E}[X_{kt}]$'s, (see (21)), in which case CbC reduces to either SDM or EBM. Finally, although from Proposition 6 we expect CbC to perform significantly better than EBM when σ_p is sufficiently low or σ_V is sufficiently high, sufficiently large patient random error (σ_p) and sufficiently low patient value heterogeneity (σ_V) are required for the utility gap to be high. This is because CbC performs close to SDM for too low σ_V or too high σ_p, and a sufficiently high σ_p and a sufficiently low σ_V are required to obtain a significant utility gap between CbC and SDM.

Third, the green regions in Figure 3 indicate the settings for which SDM yields substantially higher expected utility than CbC (the second-best policy) with an underconfident doctor (recall from Section 6 that EMB is always suboptimal for underconfident doctors). We observe that policymakers should recommend doctors to perform SDM when (i) doctors are too inaccurate and patients make moderate random errors (high σ_d and moderate σ_p; see Figure 3(a)), (ii) the medical prognoses under the two treatments are not too heterogeneous across the patient population, and patients are moderately heterogeneous in their preferences (low σ_X and moderate σ_V; see Figure 3(b)), and (iii) doctors are too underconfident (high α_p and α_d; see Figure 3(c)). In such settings, switching to SDM can result in a large increase in the expected utility, as much as 35%.

It is intuitive that doctors should not be too sophisticated in accounting for random errors (α_d and α_p should be sufficiently above 1) for SDM to outperform CbC since with sophisticated doctors, CbC is superior to SDM (recall Proposition 3(b)). On the other hand, although Proposition 6 implies that sufficiently high patient value heterogeneity, σ_V, and a sufficiently low level of patient errors, σ_p, are enough to ensure the superiority of SDM over CbC, we need to have moderate σ_p and σ_V values to obtain a significant utility gap. This is because CbC becomes equivalent to SDM for very high σ_V or very low σ_p. In addition to the need for moderate σ_p and σ_V values, prognosis variability and doctor accuracy should not be too high (sufficiently low σ_X and high σ_d) for SDM to provide significantly greater utility than CbC, contrary to expectations. The reason is as follows. With a very low prognosis variability (σ_X) or a high doctor inaccuracy (σ_d), doctors' prognosis forecasts become condensed around the mean prognosis values, $\mathbb{E}[X_{kt}]$'s, (see (21)) and the doctor under CbC either selects SDM or selects EBM for most of the cases. In fact, if the doctor is underconfident, it is very likely that the latter case will occur, i.e., CbC reduces to EBM. However, for settings with not very low patient value heterogeneity σ_V or not too high patient irrationality σ_p (as in the base case setting in our numerical study), where there is still a substantial need to incorporate patient preferences via

Recommended Decision Process	Doctor Characteristics	Patient Characteristics
CbC	sophisticated accounting for doctor and patient random errors, low doctor prognosis random error	high medical prognosis heterogeneity, low preference heterogeneity, large patient decision random error
EBM	insufficient accounting for doctor and patient random errors, high doctor prognosis random error	low medical prognosis heterogeneity, moderate preference heterogeneity, moderate patient decision random error
SDM	excessive accounting for doctor and patient random errors, high doctor prognosis random error	low medical prognosis heterogeneity, moderate preference heterogeneity, moderate patient decision random error

Table 2 Recommended decision process based on doctor and patient characteristics.

SDM, a high proportion of these selected EBM cases is indeed wrong, and this in turn worsens the performance of CbC versus SDM.

7.3. When is CbC, SDM, or EBM the right process?

Table 2 summarizes the findings from our sensitivity analyses at a high level. Policymakers and administrators should use CbC as long as doctors are sufficiently sophisticated, i.e., not too overconfident or underconfident (see Proposition 3). The advantage of CbC in such settings is greatest when patient errors are high, doctors errors are low, medical prognoses are heterogeneous, and patient preferences are homogeneous. On the other hand, when doctors are miscalibrated, policymakers and administrators may be better off promoting SDM or EBM (see Proposition 5). When doctors are overconfident, EBM is superior. When doctors are underconfident, SDM is superior. These pure policies are most important to enforce when doctors are inaccurate, patients have moderate random error, and patients are medically homogeneous but have heterogeneous preferences.

8. Conclusions

8.1. Managerial Implications

The existing medical literature has extensively explored the advantages and disadvantages of SDM, but it has given limited attention to clear guidelines on when to employ SDM. This paper offers valuable insights for policymakers, outlining when to promote SDM, discourage, or allow flexibility in implementing SDM. We find that if the doctor and patients are perfectly rational, policymakers should encourage doctors to always engage in SDM. However, we demonstrate that EBM or CbC can outperform SDM in the presence of patient and doctor random errors as well as doctors' miscalibration. This contradicts the common medical belief that SDM is always superior. We derive sufficient conditions for SDM to be the optimal policy.

Nevertheless, SDM may be easier to implement than CbC. To enhance outcomes with SDM, reducing random errors in patients and doctors is crucial. Policymakers could introduce training programs to improve patients' health literacy (Muscat et al. 2019). Alternatively, doctors could elicit patient preferences and decide on treatment by combining these preferences with their forecasts, rather than presenting forecasts and letting patients decide. To mitigate the impact of doctors' random errors, pooling forecasts from multiple doctors could harness the "wisdom of the crowd" (Surowiecki 2005, Sunstein 2006, Sjöberg 2009, Davis-Stober et al. 2014). For instance, Kattan et al. (2016) report that even averaging the judgments of as few as 5 experts in predicting the risk of positive bone scans for prostate cancer patients yielded prediction accuracy comparable to the best clinician. Group activities and team-based care, such as case conferences, expert consultation, and morning rounds, represent conventional methods to leverage the "wisdom of the crowd" (Radcliffe et al. 2019).

Lastly, we demonstrate that CbC is always the optimal policy when doctors can account for random errors effectively. To ensure the success of CbC, policymakers may organize training programs to raise doctors' awareness of patient health illiteracy and ensure that doctors communicate medical information in an understandable manner (see, e.g., the health literacy professional education and training provided by the Agency for Healthcare Research and Quality[2]). Various tools, including visual-based information, have the potential to address doctor error calibration problems and mitigate base-rate neglect (Roy and Lerch 1996, Ohlert and Weißenberger 2015).

8.2. Limitations and Future Directions

This study has several limitations, which can serve as potential directions for future research. First, our examination of the SDM process primarily involves the doctor acting as a "technical expert," providing patients with relevant information, and allowing patients to make the final treatment decision. However, SDM can be implemented in various ways. For example, the doctor may predict treatment prognoses and then elicit patient preferences to guide the treatment choice. While our focus in the present paper is to capture the core elements of a "co-production" process between doctors and patients, future work could compare different SDM approaches and identify the optimal design for the SDM process.

Second, in our comparison of SDM and EBM performance, we do not account for the time disadvantage associated with SDM. In reality, engaging in SDM typically consumes more time for doctors compared to simply following EBM recommendations. Future research could explore the time aspects of SDM and its impact on clinical practice.

Third, our analysis considers treatment prognoses under two options along only one attribute, assuming a linear utility model. Future work may delve into scenarios involving a non-linear, multi-attribute utility model and offer a more comprehensive perspective on SDM.

References

ACA (2010) Patient protection and affordable care act. *U.S. Public Law* .

Ahn JH, Hornberger JC (1996) Involving patients in the cadaveric kidney transplant allocation process: A decision-theoretic perspective. *Management Science* 42(5):629–641.

[2] https://www.ahrq.gov/health-literacy/professional-training/informed-choice.html

Alameddine M, AlGurg R, Otaki F, Alsheikh-Ali AA (2020) Physicians' perspective on shared decision-making in dubai: a cross-sectional study. *Human Resources for Health* 18(1):1–9.

Anderson SP, De Palma A, Thisse JF (1992) *Discrete choice theory of product differentiation* (MIT press).

Ayvaci MUS, Alagoz O, Ahsen ME, Burnside ES (2018) Preference-sensitive management of post-mammography decisions in breast cancer diagnosis. *Production and operations management* 27(12):2313–2338.

Baghus A, Giroldi E, Timmerman A, Schmitz E, Erkan F, Röhlinger D, Pieterse A, Dielissen P, Kramer A, Rietmeijer C, et al. (2022) Identifying residents' educational needs to optimising postgraduate medical education about shared decision-making. *Patient Education and Counseling* 105(10):3086–3095.

Bagshaw HP, Martinez A, Heidari N, Scheinker D, Pollack A, Stoyanova R, Horwitz E, Morton G, Kishan AU, Buyyounouski MK (2021) A personalized decision aid for prostate cancer shared decision making. *BMC medical informatics and decision making* 21(1):1–8.

Bar-Hillel M (1980) The base-rate fallacy in probability judgments. *Acta Psychologica* 44(3):211–233.

Barker C, Dunn S, Moore GP, Reszel J, Lemyre B, Daboval T (2019) Shared decision making during antenatal counselling for anticipated extremely preterm birth. *Paediatrics & Child Health* 24(4):240–249.

Batun S, Schaefer AJ, Bhandari A, Roberts MS (2018) Optimal liver acceptance for risk-sensitive patients. *Service Science* 10(3):320–333.

Braddock III CH, Edwards KA, Hasenberg NM, Laidley TL, Levinson W (1999) Informed decision making in outpatient practice: time to get back to basics. *Jama* 282(24):2313–2320.

Breen KJ, Cordner SM, Thomson Colin J, Plueckhahsin V (2010) *Good medical practice* (Citeseer).

Couët N, Desroches S, Robitaille H, Vaillancourt H, Leblanc A, Turcotte S, Elwyn G, Légaré F (2015) Assessments of the extent to which health-care providers involve patients in decision making: a systematic review of studies using the option instrument. *Health Expectations* 18(4):542–561.

Coulter A, Collins A (2011) Making shared decision-making a reality. *London: King's Fund* 621.

Daack-Hirsch S, Campbell CA (2014) The role of patient engagement in personalized healthcare. *Personalized Medicine* 11(1):1–4.

Davis AM, Katok E, Kwasnica AM (2014) Should sellers prefer auctions? a laboratory comparison of auctions and sequential mechanisms. *Management Science* 60(4):990–1008.

Davis-Stober CP, Budescu DV, Dana J, Broomell SB (2014) When is a crowd wise? *Decision* 1(2):79.

Daw A, Castellanos A, Yom-Tov GB, Pender J, Gruendlinger L (2020) The co-production of service: modeling service times in contact centers using hawkes processes. *arXiv preprint arXiv:2004.07861* .

de Leon J (2012) Evidence-based medicine versus personalized medicine: are they enemies? *Journal of clinical psychopharmacology* 32(2):153–164.

Edwards W (1968) Conservatism in human information processing. *Formal representation of human judgment* .

Feiler D, Tong J (2022) From noise to bias: Overconfidence in new product forecasting. *Management Science* 68(6):4685–4702.

Fuchs VR, et al. (1968) The service economy. *NBER Books* .

Gigerenzer G, Muir Gray JA (2011) Launching the century of the patient. *Better doctors, better patients, better decisions: Envisioning health care 2020*, 3–28 (MIT Press).

Haines S, Savic M, Carter A (2019) Advancing medicine ethically: Important considerations for innovative practice. *The American Journal of Bioethics* 19(6):38–40.

Ho TH, Zhang J (2008) Designing pricing contracts for boundedly rational customers: Does the framing of the fixed fee matter? *Management Science* 54(4):686–700.

Huang T, Allon G, Bassamboo A (2013) Bounded rationality in service systems. *Manufacturing & Service Operations Management* 15(2):263–279.

Ibrahim R, Kim SH, Tong J (2021) Eliciting human judgment for prediction algorithms. *Management Science* 67(4):2314–2325.

JM CC (2018) Evidence-based medicine or patient-centered medicine, or both? *Archivos argentinos de pediatría* 116(2):90–91.

Juslin P, Winman A, Hansson P (2007) The naïve intuitive statistician: A naïve sampling model of intuitive confidence intervals. *Psychological review* 114(3):678.

Kahneman D, Rosenfield A, Gandhi L, Blaser T (2016) Noise: How to overcome the high. *Hidden Cost of Inconsistent Decision Making. https. hbr. org/2016/10/noise* .

Kahneman D, Tversky A (1973) On the psychology of prediction. *Psychological review* 80(4):237.

Kalkanci B, Chen KY, Erhun F (2011) Contract complexity and performance under asymmetric demand information: An experimental evaluation. *Management science* 57(4):689–704.

Kattan MW, O'Rourke C, Yu C, Chagin K (2016) The wisdom of crowds of doctors: Their average predictions outperform their individual ones. *Medical Decision Making* 36(4):536–540.

Kremer M, Siemsen E, Thomas DJ (2016) The sum and its parts: Judgmental hierarchical forecasting. *Management Science* 62(9):2745–2764.

Li W, Denton BT, Morgan TM (2023) Optimizing active surveillance for prostate cancer using partially observable markov decision processes. *European Journal of Operational Research* 305(1):386–399.

McCaffery KJ, Smith SK, Wolf M (2010) The challenge of shared decision making among patients with lower literacy: a framework for research and development. *Medical Decision Making* 30(1):35–44.

Merchant FM, Dickert NW, Howard DH (2018) Mandatory shared decision making by the centers for medicare & medicaid services for cardiovascular procedures and other tests. *Jama* 320(7):641–642.

Moore DA, Healy PJ (2008) The trouble with overconfidence. *Psychological review* 115(2):502.

Muscat DM, Morony S, Trevena L, Hayen A, Shepherd HL, Smith SK, Dhillon HM, Luxford K, Nutbeam D, McCaffery KJ (2019) Skills for shared decision-making: evaluation of a health literacy program for consumers with lower literacy levels. *HLRP: Health Literacy Research and Practice* 3(3):S58–S74.

Ng CJ, Lee PY (2021) Practising shared decision making in primary care. *Malaysian Family Physician: the Official Journal of the Academy of Family Physicians of Malaysia* 16(1):2.

NHS England and NHS Improvement (2019) Nhs england and nhs improvement. shared decision making: Summary guide. URL https://www.england.nhs.uk/wp-content/uploads/2019/01/shared-decision-making-summary-guide-v1.pdf.

Ohlert CR, Weißenberger BE (2015) Beating the base-rate fallacy: An experimental approach on the effectiveness of different information presentation formats. *Journal of Management Control* 26(1):51–80.

Oshima Lee E, Emanuel EJ (2013) Shared decision making to improve care and reduce costs. *New England Journal of Medicine* 368(1):6–8.

Palumbo R, Manna R (2018) What if things go wrong in co-producing health services? exploring the implementation problems of health care co-production. *Policy and Society* 37(3):368–385.

Purnell TS, Joy S, Little E, Bridges JF, Maruthur N (2014) Patient preferences for noninsulin diabetes medications: a systematic review. *Diabetes care* 37(7):2055–2062.

Radcliffe K, Lyson HC, Barr-Walker J, Sarkar U (2019) Collective intelligence in medical decision-making: a systematic scoping review. *BMC medical informatics and decision making* 19(1):1–11.

Rahmani M, Roels G, Karmarkar US (2017) Collaborative work dynamics in projects with co-production. *Production and Operations Management* 26(4):686–703.

Ren Y, Croson R (2013) Overconfidence in newsvendor orders: An experimental study. *Management Science* 59(11):2502–2517.

Roels G (2014) Optimal design of coproductive services: Interaction and work allocation. *Manufacturing & Service Operations Management* 16(4):578–594.

Romana HW (2006) Is evidence-based medicine patient-centered and is patient-centered care evidence-based? *Health services research* 41(1):1.

Roy MC, Lerch FJ (1996) Overcoming ineffective mental representations in base-rate problems. *Information Systems Research* 7(2):233–247.

Salzburg Global Seminar (2011) Salzburg statement on shared decision making. *Bmj* 342.

Sampson SE, Froehle CM (2006) Foundations and implications of a proposed unified services theory. *Production and operations management* 15(2):329–343.

Schoenfeld EM, Goff SL, Elia TR, Khordipour ER, Poronsky KE, Nault KA, Lindenauer PK, Mazor KM (2018) A qualitative analysis of attending physicians' use of shared decision-making: implications for resident education. *Journal of graduate medical education* 10(1):43–50.

Schoenfeld EM, Goff SL, Elia TR, Khordipour ER, Poronsky KE, Nault KA, Lindenauer PK, Mazor KM (2019) Physician-identified barriers to and facilitators of shared decision-making in the emergency department: an exploratory analysis. *Emergency Medicine Journal* 36(6):346–354.

Shay LA, Lafata JE (2015) Where is the evidence? a systematic review of shared decision making and patient outcomes. *Medical Decision Making* 35(1):114–131.

Shippee N, Allen S, Leppin A, May C, Montori VM (2015) Attaining minimally disruptive medicine: context, challenges and a roadmap for implementation. *The journal of the Royal College of Physicians of Edinburgh* 45(2):118–122.

Simmons L, Leavitt L, Ray A, Fosburgh B, Sepucha K (2016) Shared decision making in common chronic conditions: impact of a resident training workshop. *Teaching and Learning in Medicine* 28(2):202–209.

Simon HA (1957) Models of man; social and rational. .

Sjöberg L (2009) Are all crowds equally wise? a comparison of political election forecasts by experts and the public. *Journal of Forecasting* 28(1):1–18.

Soll JB (1996) Determinants of overconfidence and miscalibration: The roles of random error and ecological structure. *Organizational Behavior and Human Decision Processes* 65(2):117–137.

Spatz ES, Krumholz HM, Moulton BW (2017) Prime time for shared decision making. *Jama* 317(13):1309–1310.

Stiggelbout AM, Pieterse AH, De Haes JC (2015) Shared decision making: concepts, evidence, and practice. *Patient education and counseling* 98(10):1172–1179.

Su X (2008) Bounded rationality in newsvendor models. *Manufacturing & Service Operations Management* 10(4):566–589.

Sunstein CR (2006) *Infotopia: How many minds produce knowledge* (Oxford University Press).

Surowiecki J (2005) *The wisdom of crowds* (Anchor).

Sweeney KG, MacAuley D, Gray DP (1998) Personal significance: the third dimension. *The Lancet* 351(9096):134–136.

Tong J, Feiler D (2017) A behavioral model of forecasting: Naive statistics on mental samples. *Management Science* 63(11):3609–3627.

van der Horst DE, Garvelink MM, Bos WJW, Stiggelbout AM, Pieterse AH (2022) For which decisions is shared decision making considered appropriate?–a systematic review. *Patient education and counseling* .

van Veenendaal H, van der Weijden T, Ubbink DT, Stiggelbout AM, van Mierlo LA, Hilders CG (2018) Accelerating implementation of shared decision-making in the netherlands: an exploratory investigation. *Patient education and counseling* 101(12):2097–2104.

Veroff D, Marr A, Wennberg DE (2013) Enhanced support for shared decision making reduced costs of care for patients with preference-sensitive conditions. *Health Affairs* 32(2):285–293.

Waddell A, Lennox A, Spassova G, Bragge P (2021) Barriers and facilitators to shared decision-making in hospitals from policy to practice: a systematic review. *Implementation Science* 16:1–23.

Williams MV, Davis T, Parker RM, Weiss BD, et al. (2002) The role of health literacy in patient-physician communication. *FAMILY MEDICINE-KANSAS CITY-* 34(5):383–389.

Xue M, Field JM (2008) Service coproduction with information stickiness and incomplete contracts: Implications for consulting services design. *Production and Operations Management* 17(3):357–372.

Zeuner R, Frosch DL, Kuzemchak MD, Politi MC (2015) Physicians' perceptions of shared decision-making behaviours: a qualitative study demonstrating the continued chasm between aspirations and clinical practice. *Health Expectations* 18(6):2465–2476.

APPENDIX

The appendices are organized as follows. In Appendix A, we present supplementary analysis for Section 6. In Appendix B, we present proofs for the results in the present paper. In Appendix C, we present the proofs of additional appendix lemmas used to prove the main results in the paper.

Throughout the proofs, $f_Z(z)$ and $\Phi(z)$ denote the probability density function (pdf) and the cumulative distribution function (cdf) of the standard normal distribution at point z, respectively. $f_X(x;\mu,\sigma)$ represents normal pdf with mean μ and standard deviation σ at point x. We denote $\Delta\mu_X = \mu_A - \mu_B$ and recall that $\Delta\mu_X < 0$ by design. $F_\gamma(.)$ denotes the cdf of the logistic distribution and it is equal to:

$$F_\gamma(x) := \frac{1}{1+\mathrm{e}^{-x}}, \tag{28}$$

whereas $f_\gamma(.)$ denotes the pdf of the logistic distribution and it is equal to:

$$f_\gamma(x) := \frac{\mathrm{e}^{-x}}{(1+\mathrm{e}^{-x})^2}. \tag{29}$$

Furthermore, let us define $\bar{\sigma}_1$ and $\bar{\sigma}$ as:

$$\bar{\sigma}_1 := \frac{\sigma_X^2}{\sqrt{\sigma_X^2+\sigma_d^2}}, \tag{30}$$

$$\bar{\sigma} := \frac{\sigma_X^2}{\sigma_X^2+\alpha_d\sigma_d^2}\sqrt{\sigma_X^2+\sigma_d^2}. \tag{31}$$

Now, consider the random variable V_k, which denotes the value of patient k along a particular attribute. Since we have $V_k \sim Unif[\underline{v},\bar{v}]$, $\mu_V = \frac{\underline{v}+\bar{v}}{2}$ and $\sigma_V = \frac{\bar{v}-\underline{v}}{2\sqrt{3}}$, the upper and lower bounds of the random variable V_k, $\bar{v}$ and $\underline{v}$, could be written as below in terms of μ_V and σ_V.

$$\bar{v} = \mu_V + \sqrt{3}\sigma_V, \;\; \underline{v} = \mu_V - \sqrt{3}\sigma_V. \tag{32}$$

Since $\underline{v} < 0$ holds by assumption, we need to have:

$$\sigma_V > \mu_V/\sqrt{3}. \tag{33}$$

Finally, $\mathcal{I}\{A\}$ denotes the indicator random variable associated with event A that has value 1 if event A occurs and has value 0 otherwise.

Appendix A: Supplementary Results and Analysis

A.1. Supplementary Results for Section 6

This section provides the details on the characterization of P_{SDM} given with (5).

Lemma 1 *Let doctors and patients be boundedly rational and doctors be miscalibrated. Then, for a particular patient with the signal $S_{kt} = s_{kt}$, the doctor conducts SDM if and only if*

$$s_{kA} - s_{kB} \geq -(\mu_A-\mu_B)\frac{\alpha_d\sigma_d^2}{\sigma_X^2} \;\; or \;\; s_{kA} - s_{kB} \leq -K^m\frac{\sigma_X^2+\alpha_d\sigma_d^2}{\sigma_X^2} - (\mu_A-\mu_B)\frac{\alpha_d\sigma_d^2}{\sigma_X^2} \tag{34}$$

holds, where $K^m = \alpha_p f(\sigma_V)\sigma_p$ *and* $f(\sigma_V) \geq 0$ *is a decreasing function of* σ_V *with* $\lim_{\sigma_V \to \mu_V/\sqrt{3}} f(\sigma_V) = \infty$ *and* $\lim_{\sigma_V \to \infty} f(\sigma_V) = 0$*. Then, the probability that the miscalibrated doctor conducts SDM, denoted by* P^b_{SDM}*, is equal to:*

$$P_{SDM} = \mathbb{P}_{S_{kt}}\left(S_{kA} - S_{kB} \geq -(\mu_A - \mu_B)\frac{\alpha_d \sigma_d^2}{\sigma_X^2}\right) + \mathbb{P}_{S_{kt}}\left(S_{kA} - S_{kB} \leq -K^m \frac{\sigma_X^2 + \alpha_d \sigma_d^2}{\sigma_X^2} - (\mu_A - \mu_B)\frac{\alpha_d \sigma_d^2}{\sigma_X^2}\right). \tag{35}$$

Figure 4 illustrates Lemma 1 by demonstrating the SDM and EBM decisions of a miscalibrated doctor with respect to $s_{kA} - s_{kB}$, i.e., the difference between the prognoses under treatment A and B observed through the signal. Accordingly, the doctor conducts SDM if and only if the difference between the prognoses under treatment A and B observed through the signal is sufficiently large. Namely, for a given patient, if the doctor observes, through the noisy signal, a lot higher prognosis under treatment A than B (Region 3), contrary to the average patient's prognosis difference (recall that $\Delta\mu_X < 0$), Treatment A is likely to bring higher utility than Treatment B, i.e., the recommendation of EBM, since the probability of having a patient with a positive value for the attribute is higher than 1/2 (due to $\mu_V > 0$). In this case, the doctor engages in SDM to inform the patient that their individual prognoses are substantially different from the average, which corresponds to the value of personalized medicine.

Similarly (perhaps, less intuitively), for a given patient, if the doctor observes, through the noisy signal, a lot lower prognosis under treatment A than B, and the two treatments perform differently (Region 1), the doctor's perceived expected utility for SDM is higher than that for EBM. The net perceived expected benefit of SDM over EBM in this case, indeed, is affected by two opposing forces, namely the value of preference integration through SDM and the risk of selecting the wrong treatment in SDM due to patient errors, and in Region 1, the first one dominates the second one. That is, because the two treatments perform differently, the probability that the erroneous patient in SDM—by messing up the forecasts—selects the treatment with a lower utility is low, whereas following EBM (equivalently pursuing treatment B) might lead to a significant utility loss if the realization of the patient's value for the attribute is negative—making crucial to let the patient incorporate their preferences.

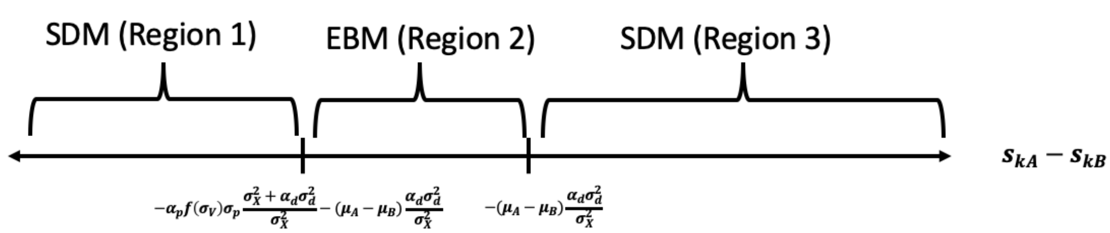


Figure 4 SDM and EBM decisions of the miscalibrated doctor

Next, using the real expected utilities for SDM and EBM in (16) and (17), in the following lemma, we characterize the optimal SDM/EBM decisions that should have been made under CbC with the miscalibrated physician's forecasts given the signal $S_{kt} = s_{kt}$.

Lemma 2 *Let doctors and patients be boundedly rational and doctors be miscalibrated. Furthermore, let* $K = f(\sigma_V)\sigma_p$*, where* $f(\sigma_V) \geq 0$ *is a decreasing function of* σ_V *with* $\lim_{\sigma_V \to \mu_V/\sqrt{3}} f(\sigma_V) = \infty$ *and* $\lim_{\sigma_V \to \infty} f(\sigma_V) = 0$*. Then, under CbC, the following statements hold:*

(a) If the doctor is overconfident ($\alpha_d < 1$, $\alpha_p < 1$)*, for a particular patient with the signal* $S_{kt} = s_{kt}$*, the doctor should conduct SDM if and only if*

$$s_{kA} - s_{kB} \geq -(\mu_A - \mu_B)\frac{\sigma_d^2}{\sigma_X^2} \text{ or } s_{kA} - s_{kB} \leq -K\frac{\sigma_X^2 + \alpha_d\sigma_d^2}{\sigma_X^2} - (\mu_A - \mu_B)\frac{\alpha_d\sigma_d^2}{\sigma_X^2} \tag{36}$$

holds.

(b) Let the doctor be underconfident ($\alpha_d > 1$, $\alpha_p > 1$)*. Then:*

(i) If $K \geq -(\mu_A - \mu_B)\frac{\sigma_d^2(\alpha_d - 1)}{\sigma_X^2 + \alpha_d\sigma_d^2}$*, the doctor should conduct SDM for a particular patient with the signal* $S_{kt} = s_{kt}$ *if and only if (36) holds.*

(ii) If $K < -(\mu_A - \mu_B)\frac{\sigma_d^2(\alpha_d - 1)}{\sigma_X^2 + \alpha_d\sigma_d^2}$*, the doctor should conduct SDM for a particular patient with the signal* $S_{kt} = s_{kt}$ *if and only if*

$$s_{kA} - s_{kB} \leq -(\mu_A - \mu_B)\frac{\sigma_d^2}{\sigma_X^2} \text{ or } s_{kA} - s_{kB} \geq -K\frac{\sigma_X^2 + \alpha_d\sigma_d^2}{\sigma_X^2} - (\mu_A - \mu_B)\frac{\alpha_d\sigma_d^2}{\sigma_X^2} \tag{37}$$

holds.

Proofs for the Supplementary Results in Appendix A.1.

For the proof of the lemmas in Appendix A.1, we need the following lemma, which we prove in Appendix C.

Lemma A1 *Let us define the function* $H(y)$ *for* $y \in (-\infty, \infty)$*:*

$$H(y) := \mathbb{E}_{V_k}\left[V_k F_\gamma\left(\frac{V_k y}{\alpha_p\sigma_p}\right)\right], \tag{38}$$

where $F_\gamma(.)$ *is introduced in (28).*

1. *There exists a unique* $y^* < 0$ *satisfying* $H(y) = \mathbb{E}_{V_k}\left[V_k F_\gamma\left(\frac{V_k y}{\alpha_p\sigma_p}\right)\right] = 0$ *such that* $H(y) \geq 0$ *if* $y \geq y^*$*, and* $H(y) < 0$ *if* $y < y^*$*.*
2. y^* *is linear and it decreases in* σ_p *and* α_p*, and increases in* σ_V*. Hence, we have* $y^* = -\alpha_p f(\sigma_V)\sigma_p$*, where* $f(\sigma_V) \geq 0$ *is a decreasing function of* σ_V *with* $\lim_{\sigma_V \to \mu_V/\sqrt{3}} f(\sigma_V) = \infty$ *and* $\lim_{\sigma_V \to \infty} f(\sigma_V) = 0$*.*

Now, we are ready to prove Lemmas 1 and 2 using Lemma A1.

Proof of Lemma 1: First, consider $H(y)$ introduced in (38). By (22), given the signal $S_{kt} = s_{kt}$, the forecast of the miscalibrated doctor for $x_{kA} - x_{kB}$ is equal to:

$$\hat{x}^m_{kA} - \hat{x}^m_{kB} = \frac{\sigma_X^2}{\sigma_X^2 + \alpha_d \sigma_d^2}(s_{kA} - s_{kB}) + \frac{\alpha_d \sigma_d^2}{\sigma_X^2 + \alpha_d \sigma_d^2}(\mu_A - \mu_B). \tag{39}$$

Now, recall from Section 6 that the expected utility difference between SDM and EBM predicted by the miscalibrated doctor for a particular patient k with $\hat{X}^m_{kt} = \hat{x}^m_{kt}$ are given by:

$$U^{SDM}(\hat{x}^m_{kt}) - U^{EBM}(\hat{x}^m_{kt}) = \mathbb{E}_{V_k}\left[V_k(\hat{x}^m_{kA} - \hat{x}^m_{kB}) P^m(V_k, \hat{x}^m_{kt})\right]. \tag{40}$$

The miscalibrated doctor, upon observing the signal $S_{kt} = s_{kt}$, conducts SDM for a particular patient k if and only if $U^{SDM}(\hat{x}^m_{kt}) - U^{EBM}(\hat{x}^m_{kt}) \geq 0$ holds. By the definition of $H(y)$ in (38), $U^{SDM}(\hat{x}^m_{kt}) - U^{EBM}(\hat{x}^m_{kt})$ is equal to:

$$U^{SDM}(\hat{x}^m_{kt}) - U^{EBM}(\hat{x}^m_{kt}) = (\hat{x}^m_{kA} - \hat{x}^m_{kB}) H(\hat{x}^m_{kA} - \hat{x}^m_{kB}), \tag{41}$$

Now, in (41), we will separately investigate the sign of $\hat{x}^m_{kA} - \hat{x}^m_{kB}$ and $H(\hat{x}^m_{kA} - \hat{x}^m_{kB})$. Using Lemma A1 and letting $K^m = \alpha_p f(\sigma_V)\sigma_p$ (see Lemma A1 and note that $-K^m$ here corresponds to y^* there), we have:

$$H(-K^m) = \mathbb{E}_{V_k}\left[V_k F_\gamma\left(\frac{-V_k K^m}{\alpha_p \sigma_p}\right)\right] = 0, \tag{42}$$

$$\begin{aligned} H(\hat{x}^m_{kA} - \hat{x}^m_{kB}) &= \mathbb{E}_{V_k}\left[V_k F_\gamma\left(\frac{V_k(\hat{x}^m_{kA} - \hat{x}^m_{kB})}{\alpha_p \sigma_p}\right)\right] > 0; \text{ for all } \hat{x}^m_{kt} \text{ with } \hat{x}^m_{kA} - \hat{x}^m_{kB} > -K^m, \text{ and} \\ H(\hat{x}^m_{kA} - \hat{x}^m_{kB}) &= \mathbb{E}_{V_k}\left[V_k F_\gamma\left(\frac{V_k(\hat{x}^m_{kA} - \hat{x}^m_{kB})}{\alpha_p \sigma_p}\right)\right] < 0; \text{ for all } \hat{x}^m_{kt} \text{ with } \hat{x}^m_{kA} - \hat{x}^m_{kB} < -K^m. \end{aligned} \tag{43}$$

Finally, using (41), (43) and accounting for the sign of $\hat{x}^m_{kA} - \hat{x}^m_{kB}$, we can conclude that

$$\begin{aligned} U^{SDM}(\hat{x}^m_{kt}) - U^{EBM}(\hat{x}^m_{kt}) &= \mathbb{E}_{V_k}\left[V_k(\hat{x}^m_{kA} - \hat{x}^m_{kB}) F_\gamma\left(\frac{V_k(\hat{x}^m_{kA} - \hat{x}^m_{kB})}{\alpha_p \sigma_p}\right)\right] \geq 0 \text{ for all } \hat{x}^m_{kt} \\ &\qquad \text{with } \hat{x}^m_{kA} - \hat{x}^m_{kB} \geq 0 \text{ or } \hat{x}^m_{kA} - \hat{x}^m_{kB} \leq -K^m; \text{ and} \end{aligned} \tag{44}$$

$$\begin{aligned} U^{SDM}(\hat{x}^m_{kt}) - U^{EBM}(\hat{x}^m_{kt}) &= \mathbb{E}_{V_k}\left[V_k(\hat{x}^m_{kA} - \hat{x}^m_{kB}) F_\gamma\left(\frac{V_k(\hat{x}^m_{kA} - \hat{x}^m_{kB})}{\alpha_p \sigma_p}\right)\right] < 0 \\ &\qquad \text{for all } \hat{x}^m_{kt} \text{ with } -K^m < \hat{x}^m_{kA} - \hat{x}^m_{kB} < 0. \end{aligned} \tag{45}$$

When we replace $\hat{x}^m_{kA} - \hat{x}^m_{kB}$ in (44) and (45) with $\frac{\sigma_X^2}{\sigma_X^2 + \alpha_d \sigma_d^2}(s_{kA} - s_{kB}) + \frac{\alpha_d \sigma_d^2}{\sigma_X^2 + \alpha_d \sigma_d^2}(\mu_A - \mu_B)$ using (39), we conclude that the doctor conducts SDM for a patient (i.e., $U^{SDM}(\hat{x}^m_{kt}) - U^{EBM}(\hat{x}^m_{kt}) \geq 0$ holds) if and only if we have $s_{kA} - s_{kB} \geq -(\mu_A - \mu_B)\frac{\alpha_d \sigma_d^2}{\sigma_X^2}$ or $s_{kA} - s_{kB} \leq -K^m \frac{\sigma_X^2 + \alpha_d \sigma_d^2}{\sigma_X^2} - (\mu_A - \mu_B)\frac{\alpha_d \sigma_d^2}{\sigma_X^2}$.

□

Proof of Lemma 2: Recall from Section 6 that the expected actual utility difference between SDM and EBM for a miscalibrated doctor who shares the forecast $\hat{X}^m_{kt} = \hat{x}^m_{kt}$ upon observing the signal $S_{kt} = s_{kt}$ is equal to:

$$U_a^{SDM}(s_{kt}) - U_a^{EBM}(s_{kt}) = \mathbb{E}_{V_k}\left[V_k\left(\mathbb{E}[X_{kA} \mid S_{kA} = s_{kA}] - \mathbb{E}[X_{kB} \mid S_{kB} = s_{kB}]\right) P(V_k, \hat{x}^m_{kt})\right]$$

$$= \mathbb{E}_{V_k}\left[V_k\Big(\mathbb{E}[X_{kA} \mid S_{kA}=s_{kA}] - \mathbb{E}[X_{kB} \mid S_{kB}=s_{kB}]\Big) F_\gamma\left(\frac{V_k\left(\hat{x}^m_{kA}-\hat{x}^m_{kB}\right)}{\sigma_p}\right)\right]$$
$$= \Big(\mathbb{E}[X_{kA} \mid S_{kA}=s_{kA}] - \mathbb{E}[X_{kB} \mid S_{kB}=s_{kB}]\Big)\mathbb{E}_{V_k}\left[V_k F_\gamma\left(\frac{V_k\left(\hat{x}^m_{kA}-\hat{x}^m_{kB}\right)}{\sigma_p}\right)\right], \quad (46)$$

where $\mathbb{E}[X_{kt}|S_{kt}]$ and $\hat{X}^m_{kt}$ are given with (10) and (21), respectively. The miscalibrated doctor, upon observing the signal $S_{kt}=s_{kt}$, should conduct SDM if and only if $U^{SDM}_a(s_{kt}) - U^{EBM}_a(s_{kt}) \geq 0$ holds. Now, similar to the proof of Lemma 1, let us define the function $H(y)$ for $y \in (-\infty,\infty)$ in the following way:

$$H(y) = \mathbb{E}_{V_k}\left[V_k F_\gamma\left(\frac{V_k y}{\sigma_p}\right)\right]. \quad (47)$$

By the definition of $H(y)$ in (47), $U^{SDM}_a(s_{kt}) - U^{EBM}_a(s_{kt})$ is equal to:

$$U^{SDM}_a(s_{kt}) - U^{EBM}(s_{kt}) = \Big(\mathbb{E}[X_{kA} \mid S_{kA}=s_{kA}] - \mathbb{E}[X_{kB} \mid S_{kB}=s_{kB}]\Big) H\Big(\hat{x}^m_{kA}-\hat{x}^m_{kB}\Big). \quad (48)$$

Now, in (48), we will separately analyze the sign of $\mathbb{E}[X_{kA} \mid S_{kA}=s_{kA}] - \mathbb{E}[X_{kB} \mid S_{kB}=s_{kB}]$ and $H\Big(\hat{x}^m_{kA}-\hat{x}^m_{kB}\Big)$. Setting $\alpha_p = 1$ in Lemma A1 and letting $K = f(\sigma_V)\sigma_p$ (see Lemma A1 and note that $-K$ here corresponds to y^* there), we have:

$$\begin{aligned}
&H(-K) = \mathbb{E}_{V_k}\left[V_k F_\gamma\left(\frac{-V_k K}{\sigma_p}\right)\right] = 0,\\
&H\Big(\hat{x}^m_{kA}-\hat{x}^m_{kB}\Big) > 0;\ \text{for all } \hat{x}^m_{kt} \text{ with } \hat{x}^m_{kA}-\hat{x}^m_{kB} > -K, \text{ and}\\
&H\Big(\hat{x}^m_{kA}-\hat{x}^m_{kB}\Big) < 0;\ \text{for all } \hat{x}^m_{kt} \text{ with } \hat{x}^m_{kA}-\hat{x}^m_{kB} < -K.
\end{aligned} \quad (49)$$

When we replace $\hat{x}^m_{kA}-\hat{x}^m_{kB}$ in (49) with $\frac{\sigma_X^2}{\sigma_X^2+\alpha_d\sigma_d^2}(s_{kA}-s_{kB}) + \frac{\alpha_d\sigma_d^2}{\sigma_X^2+\alpha_d\sigma_d^2}(\mu_A-\mu_B)$ using (39), we have:

$$\begin{aligned}
&H\Big(\hat{x}^m_{kA}-\hat{x}^m_{kB}\Big) > 0;\ \text{if } s_{kA}-s_{kB} > -K\frac{\sigma_X^2+\alpha_d\sigma_d^2}{\sigma_X^2} - (\mu_A-\mu_B)\frac{\alpha_d\sigma_d^2}{\sigma_X^2},\\
&H\Big(\hat{x}^m_{kA}-\hat{x}^m_{kB}\Big) < 0;\ \text{if } s_{kA}-s_{kB} < -K\frac{\sigma_X^2+\alpha_d\sigma_d^2}{\sigma_X^2} - (\mu_A-\mu_B)\frac{\alpha_d\sigma_d^2}{\sigma_X^2},
\end{aligned} \quad (50)$$

whereas $\mathbb{E}[X_{kA} \mid S_{kA}=s_{kA}] - \mathbb{E}[X_{kB} \mid S_{kB}=s_{kB}]$ is nonnegative if and only if $s_{kA}-s_{kB} \geq -(\mu_A-\mu_B)\frac{\sigma_d^2}{\sigma_X^2}$ holds (see (11)). Next, we will analyze parts (a) and (b) of this lemma separately:

Part (a): When the doctor is overconfident ($\alpha_d < 1$, $\alpha_p < 1$), we have $-K\frac{\sigma_X^2+\alpha_d\sigma_d^2}{\sigma_X^2} - (\mu_A-\mu_B)\frac{\alpha_d\sigma_d^2}{\sigma_X^2} \leq -(\mu_A-\mu_B)\frac{\sigma_d^2}{\sigma_X^2}$. Then, using (48), (50) and accounting for the sign of $\mathbb{E}[X_{kA} \mid S_{kA}=s_{kA}] - \mathbb{E}[X_{kB} \mid S_{kB}=s_{kB}]$, we can conclude that

$$\begin{aligned}
U^{SDM}_a(s_{kt}) - U^{EBM}_a(s_{kt}) \geq 0 \ \text{ if and only if } & s_{kA}-s_{kB} \geq -(\mu_A-\mu_B)\frac{\sigma_d^2}{\sigma_X^2}, \text{ or}\\
& s_{kA}-s_{kB} \leq -K\frac{\sigma_X^2+\alpha_d\sigma_d^2}{\sigma_X^2} - (\mu_A-\mu_B)\frac{\alpha_d\sigma_d^2}{\sigma_X^2}.
\end{aligned} \quad (51)$$

Part (b)(i): When the doctor is underconfident ($\alpha_d > 1$, $\alpha_p > 1$) and $K \geq -(\mu_A - \mu_B)\frac{\sigma_d^2(\alpha_d - 1)}{\sigma_X^2 + \alpha_d \sigma_d^2}$ holds, we have $-K\frac{\sigma_X^2 + \alpha_d\sigma_d^2}{\sigma_X^2} - (\mu_A - \mu_B)\frac{\alpha_d\sigma_d^2}{\sigma_X^2} \leq -(\mu_A - \mu_B)\frac{\sigma_d^2}{\sigma_X^2}$. Then, using (48), (50) and accounting for the sign of $\mathbb{E}[X_{kA} \mid S_{kA} = s_{kA}] - \mathbb{E}[X_{kB} \mid S_{kB} = s_{kB}]$, we again obtain (51) for the sign of $U_a^{SDM}(s_{kt}) - U_a^{EBM}(s_{kt})$.

Part (b)(ii): When the doctor is underconfident ($\alpha_d > 1$, $\alpha_p > 1$) and $K < -(\mu_A - \mu_B)\frac{\sigma_d^2(\alpha_d - 1)}{\sigma_X^2 + \alpha_d \sigma_d^2}$ holds, we have $-K\frac{\sigma_X^2 + \alpha_d\sigma_d^2}{\sigma_X^2} - (\mu_A - \mu_B)\frac{\alpha_d\sigma_d^2}{\sigma_X^2} > -(\mu_A - \mu_B)\frac{\sigma_d^2}{\sigma_X^2}$. Then, using (48), (50) and accounting for the sign of $\mathbb{E}[X_{kA} \mid S_{kA} = s_{kA}] - \mathbb{E}[X_{kB} \mid S_{kB} = s_{kB}]$, we can conclude that

$$U_a^{SDM}(s_{kt}) - U_a^{EBM}(s_{kt}) \geq 0 \text{ if and only if } s_{kA} - s_{kB} \leq -(\mu_A - \mu_B)\frac{\sigma_d^2}{\sigma_X^2}, \text{ or}$$
$$s_{kA} - s_{kB} \geq -K\frac{\sigma_X^2 + \alpha_d\sigma_d^2}{\sigma_X^2} - (\mu_A - \mu_B)\frac{\alpha_d\sigma_d^2}{\sigma_X^2}. \quad (52)$$

□

Appendix B: Proofs of Results

B.1. Proof of Section 4 Results

Proof of Proposition 1: (a) Given $V_k = v_k$ and $X_{kt} = x_{kt}$ for a particular patient k, by (7) and (8), the expected utility difference between SDM and EBM (perceived by the doctor) is equal to :

$$\begin{aligned} U^{SDM}(x_{kt}) - U^{EBM}(x_{kt}) &= \mathbb{E}_{V_k}\left[V_k(x_{kA} - x_{kB})P(V_k, x_{kt})\right] \\ &= \mathbb{E}_{V_k}\left[\max\left\{V_k(x_{kA} - x_{kB}), 0\right\}\right], \end{aligned} \quad (53)$$

where $P(V_k, x_{kt})$ is given with (6) and the second equality follows from the definition of $P(V_k, x_{kt})$. Since $U^{SDM}(x_{kt}) - U^{EBM}(x_{kt})$ given with (53) is nonnegative for any x_{kt}, SDM probability of the physician under CbC is:

$$\mathbb{P}_{X_{kt}}\left(U^{SDM}(X_{kt}) - U^{EBM}(X_{kt}) \geq 0\right) = 1.$$

To that end, CbC is equivalent to SDM.

(b) Let U_d denote the expected utility difference between CbC (or, equivalently, SDM) and EBM with a perfectly rational physician and patients. By (53), U_d is equal to:

$$U_d = \mathbb{E}_{V_k, X_{kt}}\left[\max\{V_k(X_{kA} - X_{kB}), 0\}\right]. \quad (54)$$

Since $\max\{V_k(X_{kA} - X_{kB}), 0\} \geq 0$, $U_d \geq 0$ always holds.

□

B.2. Proof of Section 5 Results

Proof of Proposition 2: (a) Consider Lemma 1 with $\alpha_p = \alpha_d = 1$, which corresponds to a sophisticated doctor. It directly follows from (35) that $P_{SDM} < 1$ as long as $\sigma_p > 0$ holds.

(b) For any observed signal $S_{kt} = s_{kt}$, consider the perceived expected SDM and EBM utilities, $U^{SDM}(\hat{x}_{kt})$ and $U^{EBM}(\hat{x}_{kt})$, given with (14) and (15), as well as the actual expected SDM and EBM utilities, $U_a^{SDM}(s_{kt})$ and $U_a^{EBM}(s_{kt})$, given with (16) and (17). Noting that $\hat{x}_{kt} = \mathbb{E}[X_{kt}|S_{kt} = s_{kt}]$, it is straightforward to see that $U^{SDM}(\hat{x}_{kt}) = U_a^{SDM}(s_{kt})$ and $U^{EBM}(\hat{x}_{kt}) = U_a^{EBM}(s_{kt})$ hold for all realizations of the signal (for all $s_{kt} \in (-\infty, \infty)$). Then, by (18), (19) and (20), we have $P_W^{SDM} = P_W^{EBM} = P_W = 0$.

□

Proof of Proposition 3: First, we will introduce the expressions for the expected utilities of SDM, EBM, and CbC. The expected utility provided by SDM, denoted by EU^{SDM}, is equal to:

$$EU^{SDM} = \mathbb{E}_{V_k, X_{kt}}\left[V_k X_{kA} P\left(V_k, \hat{X}_{kt}\right) + V_k X_{kB}\left(1 - P\left(V_k, \hat{X}_{kt}\right)\right)\right], \tag{55}$$

where $\hat{X}_{kA} - \hat{X}_{kB}$ is given with (11). On the other hand, the expected utility of EBM, denoted by EU^{EBM}, is equal to:

$$EU^{EBM} = \mathbb{E}_{V_k, X_{kB}}[V_k X_{kB}]. \tag{56}$$

Furthermore, the expected utility of CbC, denoted by EU^{CBC}, is equal to:

$$\begin{aligned} EU^{CBC} = \mathbb{E}_{V_k, X_{kt}}&\left[\left(V_k X_{kA} P\left(V_k, \hat{X}_{kt}\right) + V_k X_{kB}\left(1 - P\left(V_k, \hat{X}_{kt}\right)\right)\right)\mathcal{I}\left\{U^{SDM}(\hat{X}_{kt}) - U^{EBM}(\hat{X}_{kt}) \geq 0\right\}\right] \\ &+ \mathbb{E}_{V_k, X_{kB}}\left[V_k X_{kB} \mathcal{I}\left\{U^{SDM}(\hat{X}_{kt}) - U^{EBM}(\hat{X}_{kt}) < 0\right\}\right], \end{aligned} \tag{57}$$

where $U^{SDM}(\hat{x}_{kt})$ and $U^{EBM}(\hat{x}_{kt})$ are defined in (14) and (15), whereas $\hat{X}_{kt}$ is a random variable which is a function of the signal S_{kt} and it is given with (11).

(a) Recalling that $\hat{X}_{kt} = \mathbb{E}[X_{kt} \mid S_{kt}]$ (see (10)) for a sophisticated doctor, we can write the expected utility difference between SDM and EBM, denoted by U_d^{SE}, as:

$$\begin{aligned} U_d^{SE} &= EU^{SDM} - EU^{EBM} \\ &= \mathbb{E}_{V_k, S_{kt}, X_{kt}}\left[V_k(X_{kA} - X_{kB}) P\left(V_k, \hat{X}_{kt}\right)\right] \\ &= \mathbb{E}_{V_k, S_{kt}, X_{kt}}\left[V_k(X_{kA} - X_{kB}) P\left(V_k, \mathbb{E}[X_{kt} \mid S_{kt}]\right)\right] \\ &= \mathbb{E}_{V_k, S_{kt}}\left[\mathbb{E}_{X_{kt}}\left[V_k(X_{kA} - X_{kB}) P\left(V_k, \mathbb{E}[X_{kt} \mid S_{kt}]\right) \mid V_k, S_{kt}\right]\right] \\ &= \mathbb{E}_{V_k, S_{kt}}\left[\mathbb{E}_{X_{kt}}\left[(X_{kA} - X_{kB}) \mid V_k, S_{kt}\right] V_k P\left(V_k, \mathbb{E}[X_{kt} \mid S_{kt}]\right)\right] \\ &= \mathbb{E}_{V_k, S_{kt}}\left[\left(\mathbb{E}[X_{kA} \mid S_{kA}] - \mathbb{E}[X_{kB} \mid S_{kB}]\right) V_k P\left(V_k, \mathbb{E}[X_{kt} \mid S_{kt}]\right)\right] \\ &= \mathbb{E}_{V_k, S_{kt}}\left[\left(\mathbb{E}[X_{kA} \mid S_{kA}] - \mathbb{E}[X_{kB} \mid S_{kB}]\right) V_k F_\gamma\left(\frac{V_k\left(\mathbb{E}[X_{kA} \mid S_{kA}] - \mathbb{E}[X_{kB} \mid S_{kB}]\right)}{\sigma_p}\right)\right] \\ &= \mathbb{E}_{V_k, Z}\left[V_k(\Delta\mu_X + \bar{\sigma}_1 Z) F_\gamma\left(\frac{V_k\left(\Delta\mu_X + \bar{\sigma}_1 Z\right)}{\sigma_p}\right)\right], \end{aligned} \tag{58}$$

where the subscripts in the expectations indicate what variable the expectation is taken over, and $P(V_k, \hat{x}_{kt})$ and $F_\gamma(.)$ are given with (13) and (28), respectively. The second equality follows from (55) and (56), the third equality follows from substituting $\mathbb{E}[X_{kt} \mid S_{kt}]$ for $\hat{X}_{kt}$, the fourth equality follows from the law of iterated expectations, and the fifth equality follows from the fact that when V_k and S_{kt} are given, $V_k P(V_k, \mathbb{E}[X_{kt} \mid S_{kt}])$ is not random anymore. The sixth equality follows from the independence of S_{kt} and V_k, and the seventh equality follows from replacing $P(V_k, \mathbb{E}[X_{kt} \mid S_{kt}])$ with the cdf of the standard logistic distribution, $F_\gamma(.)$, which is defined in (28). Finally, we obtain the last equality from the following argument: We can plug $\frac{\sigma_X^2}{\sigma_X^2+\sigma_d^2}(S_{kA}-S_{kB}) + \frac{\sigma_d^2}{\sigma_X^2+\sigma_d^2}\Delta\mu_X$ for $\mathbb{E}[X_{kA} \mid S_{kA}] - \mathbb{E}[X_{kB} \mid S_{kB}]$ by (11). As S_{kt} is given with (9), we have:

$$\mathbb{E}[X_{kA} \mid S_{kA}] - \mathbb{E}[X_{kB} \mid S_{kB}] = \frac{\sigma_X^2}{\sigma_X^2+\sigma_d^2}(S_{kA}-S_{kB}) + \frac{\sigma_d^2}{\sigma_X^2+\sigma_d^2}\Delta\mu_X = \Delta\mu_X + \bar{\sigma}_1 Z, \tag{59}$$

where Z is a standard normal random variable, and $\bar{\sigma}_1$ is given with (30).

Finally, we will show that U_d^{SE} becomes negative under certain conditions, e.g., when σ_p is sufficiently high. The derivative of U_d^{SE} with respect to σ_p is

$$\frac{dU_d^{SE}}{d\sigma_p} = -\mathbb{E}_{V_k,Z}\left[\frac{V_k^2}{\sigma_p^2}(\Delta\mu_X + \bar{\sigma}_1 Z)^2 f_\gamma\left(\frac{V_k(\Delta\mu_X + \bar{\sigma}_1 Z)}{\sigma_p}\right)\right],$$

where we interchange the expectation and differentiation by resting on the Dominated Convergence Theorem. Moreover, $f_\gamma(.)$ is introduced in (29) and it denotes the pdf of the standard logistic distribution. Since we have $\frac{dU_d^{SE}}{d\sigma_p} < 0$, it follows that U_d^{SE} decreases in σ_p. Furthermore, one can easily confirm that $U_d^{SE} > 0$ as $\sigma_p \to 0$, whereas $U_d^{SE} = \mu_V \Delta\mu_X/2 < 0$ as $\sigma_p \to \infty$. This shows that U_d^{SE} becomes negative under certain conditions, e.g., when σ_p is sufficiently high.

(b) *Part I:* The expected utility difference between CbC and EBM, denoted by U_d^{CE}, is equal to:

$$\begin{aligned}
&U_d^{CE} = EU^{CBC} - EU^{EBM} \\
&= \mathbb{E}_{V_k,S_{kt},X_{kt}}\left[V_k(X_{kA}-X_{kB})P\left(V_k,\hat{X}_{kt}\right)\mathcal{I}\left\{U^{SDM}(\hat{X}_{kt}) - U^{EBM}(\hat{X}_{kt}) \geq 0\right\}\right] \\
&= \mathbb{E}_{V_k,S_{kt}}\left[\mathbb{E}_{X_{kt}}\left[V_k(X_{kA}-X_{kB})P\left(V_k,\hat{X}_{kt}\right)\mathcal{I}\left\{U^{SDM}(\hat{X}_{kt}) - U^{EBM}(\hat{X}_{kt}) \geq 0\right\}\middle| V_k, S_{kt}\right]\right] \\
&= \mathbb{E}_{V_k,S_{kt}}\left[\mathbb{E}_{X_{kt}}\left[(X_{kA}-X_{kB}) \mid V_k, S_{kt}\right]V_k P\left(V_k,\hat{X}_{kt}\right)\mathcal{I}\left\{U^{SDM}(\hat{X}_{kt}) - U^{EBM}(\hat{X}_{kt}) \geq 0\right\}\right] \\
&= \mathbb{E}_{V_k,S_{kt}}\left[V_k\left(\mathbb{E}[X_{kA} \mid S_{kA}] - \mathbb{E}[X_{kB} \mid S_{kB}]\right)P\left(V_k,\hat{X}_{kt}\right)\mathcal{I}\left\{U^{SDM}(\hat{X}_{kt}) - U^{EBM}(\hat{X}_{kt}) \geq 0\right\}\right] \\
&= \mathbb{E}_{S_{kt}}\left[\mathbb{E}_{V_k}\left[V_k\left(\mathbb{E}[X_{kA} \mid S_{kA}] - \mathbb{E}[X_{kB} \mid S_{kB}]\right)P\left(V_k,\hat{X}_{kt}\right)\mathcal{I}\left\{U^{SDM}(\hat{X}_{kt}) - U^{EBM}(\hat{X}_{kt}) \geq 0\right\}\middle| S_{kt}\right]\right] \\
&= \mathbb{E}_{S_{kt}}\left[\mathbb{E}_{V_k}\left[V_k\left(\mathbb{E}[X_{kA} \mid S_{kA}] - \mathbb{E}[X_{kB} \mid S_{kB}]\right)P\left(V_k,\hat{X}_{kt}\right) \mid S_{kt}\right]\mathcal{I}\left\{U^{SDM}(\hat{X}_{kt}) - U^{EBM}(\hat{X}_{kt}) \geq 0\right\}\right] \\
&= \mathbb{E}_{S_{kt}}\left[\mathbb{E}_{V_k}\left[V_k\left(\mathbb{E}[X_{kA} \mid S_{kA}] - \mathbb{E}[X_{kB} \mid S_{kB}]\right)P\left(V_k,\hat{X}_{kt}\right)\right]\mathcal{I}\left\{U^{SDM}(\hat{X}_{kt}) - U^{EBM}(\hat{X}_{kt}) \geq 0\right\}\right] \\
&= \mathbb{E}_{S_{kt}}\left[\left(U_a^{SDM}(S_{kt}) - U_a^{EBM}(S_{kt})\right)\mathcal{I}\left\{U^{SDM}(\hat{X}_{kt}) - U^{EBM}(\hat{X}_{kt}) \geq 0\right\}\right] \\
&\geq 0,
\end{aligned} \tag{60}$$

where the second equality follows from (56) and (57), the third equality follows from the law of iterated expectations, and the fourth equality follows from the fact that when V_k and S_{kt} are given, $V_k P(V_k, \mathbb{E}[X_{kt} \mid S_{kt}]) \mathcal{I}\left\{U^{SDM}(\hat{X}_{kt}) - U^{EBM}(\hat{X}_{kt}) \geq 0\right\}$ is not random anymore. The fifth equality follows from the independence of S_{kt} and V_k, the sixth equality follows from the law of iterated expectations, and the seventh equality follows from the fact that when S_{kt} is given, $\mathcal{I}\left\{U^{SDM}(\hat{X}_{kt}) - U^{EBM}(\hat{X}_{kt}) \geq 0\right\}$ is not random anymore. Moreover, the eighth equality follows from the independence of V_k and S_{kt}, and the last equality follows from (16) and (17). Finally, the last inequality (the nonnegativity of U_d^{CE}) holds since we have $U^{SDM}(\hat{x}_{kt}) = U_a^{SDM}(s_{kt})$ and $U^{EBM}(\hat{x}_{kt}) = U_a^{EBM}(s_{kt})$ for any given signal $S_{kt} = s_{kt}$.

Part II: The expected utility difference between CbC and SDM, denoted by U_d^{CS}, is equal to:

$$\begin{aligned}
&U_d^{CS} = EU^{CBC} - EU^{SDM} \\
&= -\mathbb{E}_{V_k, S_{kt}, X_{kt}}\left[V_k(X_{kA} - X_{kB})P\left(V_k, \hat{X}_{kt}\right)\mathcal{I}\left\{U^{SDM}(\hat{X}_{kt}) - U^{EBM}(\hat{X}_{kt}) < 0\right\}\right] \\
&= -\mathbb{E}_{V_k, S_{kt}}\left[\mathbb{E}_{X_{kt}}\left[V_k(X_{kA} - X_{kB})P\left(V_k, \hat{X}_{kt}\right)\mathcal{I}\left\{U^{SDM}(\hat{X}_{kt}) - U^{EBM}(\hat{X}_{kt}) < 0\right\}\middle| V_k, S_{kt}\right]\right] \\
&= -\mathbb{E}_{V_k, S_{kt}}\left[\mathbb{E}_{X_{kt}}\left[(X_{kA} - X_{kB}) \mid V_k, S_{kt}\right] V_k P\left(V_k, \hat{X}_{kt}\right)\mathcal{I}\left\{U^{SDM}(\hat{X}_{kt}) - U^{EBM}(\hat{X}_{kt}) < 0\right\}\right] \\
&= -\mathbb{E}_{V_k, S_{kt}}\left[V_k\left(\mathbb{E}[X_{kA} \mid S_{kA}] - \mathbb{E}[X_{kB} \mid S_{kB}]\right)P\left(V_k, \hat{X}_{kt}\right)\mathcal{I}\left\{U^{SDM}(\hat{X}_{kt}) - U^{EBM}(\hat{X}_{kt}) < 0\right\}\right] \qquad (61) \\
&= -\mathbb{E}_{S_{kt}}\left[\mathbb{E}_{V_k}\left[V_k\left(\mathbb{E}[X_{kA} \mid S_{kA}] - \mathbb{E}[X_{kB} \mid S_{kB}]\right)P\left(V_k, \hat{X}_{kt}\right)\mathcal{I}\left\{U^{SDM}(\hat{X}_{kt}) - U^{EBM}(\hat{X}_{kt}) < 0\right\}\middle| S_{kt}\right]\right] \\
&= -\mathbb{E}_{S_{kt}}\left[\mathbb{E}_{V_k}\left[V_k\left(\mathbb{E}[X_{kA} \mid S_{kA}] - \mathbb{E}[X_{kB} \mid S_{kB}]\right)P\left(V_k, \hat{X}_{kt}\right) \mid S_{kt}\right]\mathcal{I}\left\{U^{SDM}(\hat{X}_{kt}) - U^{EBM}(\hat{X}_{kt}) < 0\right\}\right] \\
&= -\mathbb{E}_{S_{kt}}\left[\mathbb{E}_{V_k}\left[V_k\left(\mathbb{E}[X_{kA} \mid S_{kA}] - \mathbb{E}[X_{kB} \mid S_{kB}]\right)P\left(V_k, \hat{X}_{kt}\right)\right]\mathcal{I}\left\{U^{SDM}(\hat{X}_{kt}) - U^{EBM}(\hat{X}_{kt}) < 0\right\}\right] \\
&= -\mathbb{E}_{S_{kt}}\left[\left(U_a^{SDM}(S_{kt}) - U_a^{EBM}(S_{kt})\right)\mathcal{I}\left\{U^{SDM}(\hat{X}_{kt}) - U^{EBM}(\hat{X}_{kt}) < 0\right\}\right] \\
&\geq 0,
\end{aligned}$$

where the second equality follows from (55) and (57), the third equality follows from the law of iterated expectations, and the fourth equality follows from the fact that when V_k and S_{kt} are given, $V_k P(V_k, \mathbb{E}[X_{kt} \mid S_{kt}]) \mathcal{I}\left\{U^{SDM}(\hat{X}_{kt}) - U^{EBM}(\hat{X}_{kt}) < 0\right\}$ is not random anymore. The fifth equality follows from the independence of S_{kt} and V_k, the sixth equality follows from the law of iterated expectations, and the seventh equality follows from the fact that when S_{kt} is given, $\mathcal{I}\left\{U^{SDM}(\hat{X}_{kt}) - U^{EBM}(\hat{X}_{kt}) < 0\right\}$ is not random anymore. Moreover, the eighth equality follows from the independence of V_k and S_{kt}, and the last equality follows from (16) and (17). Finally, the last inequality (the nonnegativity of U_d^{CS}) holds since we have $U^{SDM}(\hat{x}_{kt}) = U_a^{SDM}(s_{kt})$ and $U^{EBM}(\hat{x}_{kt}) = U_a^{EBM}(s_{kt})$ for any given signal $S_{kt} = s_{kt}$.

□

B.3. Proof of Section 6 Results

Proof of Proposition 4: *Part (a)* It directly follows from Lemma 1 and Lemma 2(a) that the probability of wrong SDM, probability of wrong EBM, and probability of following the "wrong" strategy, i.e., P_W^{SDM}, P_W^{EBM}, and P_W, are equal to:

$$P_W^{EBM} = 0,$$
$$P_W^{SDM} = \mathbb{P}_{S_{kt}}\left(-(\mu_A-\mu_B)\frac{\alpha_d\sigma_d^2}{\sigma_X^2} \le S_{kA}-S_{kB} \le -(\mu_A-\mu_B)\frac{\sigma_d^2}{\sigma_X^2}\right)$$
$$+\mathbb{P}_{S_{kt}}\left(-K\frac{\sigma_X^2+\alpha_d\sigma_d^2}{\sigma_X^2}-(\mu_A-\mu_B)\frac{\alpha_d\sigma_d^2}{\sigma_X^2} \le S_{kA}-S_{kB} \le -K^m\frac{\sigma_X^2+\alpha_d\sigma_d^2}{\sigma_X^2}-(\mu_A-\mu_B)\frac{\alpha_d\sigma_d^2}{\sigma_X^2}\right).$$

Part (b)(i): First, using Lemma 1 and Lemma 2, let us set $K=\sigma_p f(\sigma_V)$ and $K^m=\alpha_p\sigma_p f(\sigma_V)$, where $f(\sigma_V)$ is introduced in Lemma 1 and Lemma 2 and it is a positive and decreasing function of σ_V. Furthermore, let $\Phi = -\frac{\mu_A-\mu_B}{f(\sigma_V)}\frac{\sigma_d^2(\alpha_d-1)}{\sigma_X^2+\alpha_d\sigma_d^2}$. We have two subcases:
Case I: $\alpha_p\sigma_p \ge \sigma_p \ge \Phi \Leftrightarrow K^m \ge K \ge -(\mu_A-\mu_B)\frac{\sigma_d^2(\alpha_d-1)}{\sigma_X^2+\alpha_d\sigma_d^2}$. It directly follows from Lemma 1 and Lemma 2(b)(i) that the probability of wrong SDM, and the probability of wrong EBM, i.e., P_W^{SDM} and P_W^{EBM}, are equal to:

$$P_W^{EBM} = \mathbb{P}_{S_{kt}}\left(-(\mu_A-\mu_B)\frac{\sigma_d^2}{\sigma_X^2} \le S_{kA}-S_{kB} \le -(\mu_A-\mu_B)\frac{\alpha_d\sigma_d^2}{\sigma_X^2}\right)$$
$$+\mathbb{P}_{S_{kt}}\left(-K^m\frac{\sigma_X^2+\alpha_d\sigma_d^2}{\sigma_X^2}-(\mu_A-\mu_B)\frac{\alpha_d\sigma_d^2}{\sigma_X^2} \le S_{kA}-S_{kB} \le -K\frac{\sigma_X^2+\alpha_d\sigma_d^2}{\sigma_X^2}-(\mu_A-\mu_B)\frac{\alpha_d\sigma_d^2}{\sigma_X^2}\right),$$
$$P_W^{SDM} = 0.$$

Case II: $\alpha_p\sigma_p \ge \Phi \ge \sigma_p \Leftrightarrow K^m \ge -(\mu_A-\mu_B)\frac{\sigma_d^2(\alpha_d-1)}{\sigma_X^2+\alpha_d\sigma_d^2} \ge K$. It directly follows from Lemma 1 and Lemma 2(b)(ii) that the probability of wrong SDM, and the probability of wrong EBM, i.e., P_W^{SDM} and P_W^{EBM}, are equal to:

$$P_W^{EBM} = \mathbb{P}_{S_{kt}}\left(-K^m\frac{\sigma_X^2+\alpha_d\sigma_d^2}{\sigma_X^2}-(\mu_A-\mu_B)\frac{\alpha_d\sigma_d^2}{\sigma_X^2} \le S_{kA}-S_{kB} \le -(\mu_A-\mu_B)\frac{\sigma_d^2}{\sigma_X^2}\right)$$
$$+\mathbb{P}_{S_{kt}}\left(-K\frac{\sigma_X^2+\alpha_d\sigma_d^2}{\sigma_X^2}-(\mu_A-\mu_B)\frac{\alpha_d\sigma_d^2}{\sigma_X^2} \le S_{kA}-S_{kB} \le -(\mu_A-\mu_B)\frac{\alpha_d\sigma_d^2}{\sigma_X^2}\right),$$
$$P_W^{SDM} = 0.$$

Part (b)(ii): Since $\alpha_p\sigma_p < \Phi \Leftrightarrow K^m < -(\mu_A-\mu_B)\frac{\sigma_d^2(\alpha_d-1)}{\sigma_X^2+\alpha_d\sigma_d^2}$ holds, we have $K < K^m < -(\mu_A-\mu_B)\frac{\sigma_d^2(\alpha_d-1)}{\sigma_X^2+\alpha_d\sigma_d^2}$. It directly follows from Lemma 1 and Lemma 2(b)(ii) that the probability of wrong SDM and the probability of wrong EBM, i.e., P_W^{SDM}, and P_W^{EBM}, are equal to:

$$P_W^{EBM} = \mathbb{P}_{S_{kt}}\left(-K\frac{\sigma_X^2+\alpha_d\sigma_d^2}{\sigma_X^2}-(\mu_A-\mu_B)\frac{\alpha_d\sigma_d^2}{\sigma_X^2} \le S_{kA}-S_{kB} \le -(\mu_A-\mu_B)\frac{\alpha_d\sigma_d^2}{\sigma_X^2}\right),$$
$$P_W^{SDM} = \mathbb{P}_{S_{kt}}\left(-(\mu_A-\mu_B)\frac{\sigma_d^2}{\sigma_X^2} \le S_{kA}-S_{kB} \le -K^m\frac{\sigma_X^2+\alpha_d\sigma_d^2}{\sigma_X^2}-(\mu_A-\mu_B)\frac{\alpha_d\sigma_d^2}{\sigma_X^2}\right). \quad (62)$$

Furthermore, by (35) and (62), $P_{SDM} - P_W^{SDM}$ is equal to:

$$P_{SDM} - P_W^{SDM} = \mathbb{P}_{S_{kt}}\left(S_{kA} - S_{kB} \le -(\mu_A - \mu_B)\frac{\sigma_d^2}{\sigma_X^2}\right) + \mathbb{P}_{S_{kt}}\left(S_{kA} - S_{kB} \ge -(\mu_A - \mu_B)\frac{\alpha_d\sigma_d^2}{\sigma_X^2}\right).$$

Finally, noting that $S_{kA} - S_{kB}$ is normally distributed with mean $\mu_A - \mu_B$ and variance $\sigma_X^2 + \sigma_d^2$, we have the following inequalities for P_W^{SDM} and $P_{SDM} - P_W^{SDM}$:

$$\begin{aligned}
P_W^{SDM} &= \mathbb{P}_{S_{kt}}\left(S_{kA} - S_{kB} \le -K^m\frac{\sigma_X^2 + \alpha_d\sigma_d^2}{\sigma_X^2} - (\mu_A - \mu_B)\frac{\alpha_d\sigma_d^2}{\sigma_X^2}\right) - \mathbb{P}_{S_{kt}}\left(S_{kA} - S_{kB} \le -(\mu_A - \mu_B)\frac{\sigma_d^2}{\sigma_X^2}\right)\\
&= 1 - \mathbb{P}_{S_{kt}}\left(S_{kA} - S_{kB} \ge -K^m\frac{\sigma_X^2 + \alpha_d\sigma_d^2}{\sigma_X^2} - (\mu_A - \mu_B)\frac{\alpha_d\sigma_d^2}{\sigma_X^2}\right) - \left(1 - \mathbb{P}_{S_{kt}}\left(S_{kA} - S_{kB} \ge -(\mu_A - \mu_B)\frac{\sigma_d^2}{\sigma_X^2}\right)\right)\\
&= \mathbb{P}_{S_{kt}}\left(S_{kA} - S_{kB} \ge -(\mu_A - \mu_B)\frac{\sigma_d^2}{\sigma_X^2}\right) - \mathbb{P}_{S_{kt}}\left(S_{kA} - S_{kB} \ge -K^m\frac{\sigma_X^2 + \alpha_d\sigma_d^2}{\sigma_X^2} - (\mu_A - \mu_B)\frac{\alpha_d\sigma_d^2}{\sigma_X^2}\right)\\
&\le \mathbb{P}_{S_{kt}}\left(S_{kA} - S_{kB} \ge -(\mu_A - \mu_B)\frac{\sigma_d^2}{\sigma_X^2}\right)\\
&= 1 - \mathbb{P}_{S_{kt}}\left(S_{kA} - S_{kB} \le -(\mu_A - \mu_B)\frac{\sigma_d^2}{\sigma_X^2}\right)\\
&\le \mathbb{P}_{S_{kt}}\left(S_{kA} - S_{kB} \le -(\mu_A - \mu_B)\frac{\sigma_d^2}{\sigma_X^2}\right)\\
&\le \mathbb{P}_{S_{kt}}\left(S_{kA} - S_{kB} \le -(\mu_A - \mu_B)\frac{\sigma_d^2}{\sigma_X^2}\right) + \mathbb{P}_{S_{kt}}\left(S_{kA} - S_{kB} \ge -(\mu_A - \mu_B)\frac{\alpha_d\sigma_d^2}{\sigma_X^2}\right)\\
&= P_{SDM} - P_W^{SDM},
\end{aligned}$$

where the second inequality follows from $\mathbb{P}_{S_{kt}}\left(S_{kA} - S_{kB} \le -(\mu_A - \mu_B)\frac{\sigma_d^2}{\sigma_X^2}\right) \le 1/2$ recalling that $S_{kA} - S_{kB}$ is normally distributed with mean $\mu_A - \mu_B$ and variance $\sigma_X^2 + \sigma_d^2$, and the rest follows from the algebra.

□

Proof of Proposition 5: Replacing $\hat{X}_{kt}$ with $\hat{X}_{kt}^m$ in (55), (56) and (57), we obtain the expected utility for SDM, EBM and CbC for the miscalibrated doctor.

(a) In this part, we consider a miscalibrated doctor who is not underconfident, i.e., $\alpha_p, \alpha_d \le 1$. First, note that using (11) and (22), $\mathbb{E}[X_{kA} \mid S_{kA}] - \mathbb{E}[X_{kB} \mid S_{kB}]$ could be written in terms of $\hat{X}_{kA}^m - \hat{X}_{kB}^m$ as follows:

$$\mathbb{E}[X_{kA} \mid S_{kA}] - \mathbb{E}[X_{kB} \mid S_{kB}] = \frac{\sigma_X^2 + \alpha_d\sigma_d^2}{\sigma_X^2 + \sigma_d^2}(\hat{X}_{kA}^m - \hat{X}_{kB}^m) + \frac{(1-\alpha_d)\sigma_d^2}{\sigma_X^2 + \sigma_d^2}\Delta\mu_X. \tag{63}$$

Furthermore, the following inequalities hold by (43) and (45) in Lemma 1 when we let $\alpha_p = 1$:

$$\mathbb{E}_{V_k}\left[V_k F_\gamma\left(\frac{V_k(\hat{x}_{kA}^m - \hat{x}_{kB}^m)}{\sigma_p}\right)\right] > 0; \text{ for all } \hat{x}_{kt}^m \text{ with } \hat{x}_{kA}^m - \hat{x}_{kB}^m > -K, \text{ and} \tag{64}$$

$$\mathbb{E}_{V_k}\left[V_k(\hat{x}_{kA}^m - \hat{x}_{kB}^m) F_\gamma\left(\frac{V_k(\hat{x}_{kA}^m - \hat{x}_{kB}^m)}{\sigma_p}\right)\right] < 0; \text{ for all } \hat{x}_{kt}^m \text{ with } -K < \hat{x}_{kA}^m - \hat{x}_{kB}^m < 0, \tag{65}$$

where $K = \sigma_p f(\sigma_V)$. Replacing $\hat{X}_{kt}$ with $\hat{X}_{kt}^m$ in (61), the expected utility difference between CbC and SDM for the miscalibrated doctor could be written as follows:

$$U_d^{CS} = -\mathbb{E}_{V_k, S_{kt}}\left[V_k\left(\mathbb{E}[X_{kA} \mid S_{kA}] - \mathbb{E}[X_{kB} \mid S_{kB}]\right) P\left(V_k, \hat{X}_{kt}^m\right) \mathcal{I}\left\{U^{SDM}(\hat{X}_{kt}^m) - U^{EBM}(\hat{X}_{kt}^m) < 0\right\}\right]$$

$$= -\mathbb{E}_{V_k,S_{kt}}\left[V_k\left(\mathbb{E}[X_{kA}\mid S_{kA}]-\mathbb{E}[X_{kB}\mid S_{kB}]\right)F_\gamma\left(\frac{V_k\left(\hat{X}^m_{kA}-\hat{X}^m_{kB}\right)}{\sigma_p}\right)\right.$$
$$\left.\mathcal{I}\left\{U^{SDM}(\hat{X}^m_{kt})-U^{EBM}(\hat{X}^m_{kt})<0\right\}\right]$$
$$= -\mathbb{E}_{V_k,S_{kt}}\left[V_k\left(\mathbb{E}[X_{kA}\mid S_{kA}]-\mathbb{E}[X_{kB}\mid S_{kB}]\right)F_\gamma\left(\frac{V_k\left(\hat{X}^m_{kA}-\hat{X}^m_{kB}\right)}{\sigma_p}\right)\mathcal{I}\left\{-K^m<\hat{X}^m_{kA}-\hat{X}^m_{kB}<0\right\}\right]$$
$$= -\mathbb{E}_{V_k,S_{kt}}\left[V_k\left(\frac{\sigma_X^2+\alpha_d\sigma_d^2}{\sigma_X^2+\sigma_d^2}(\hat{X}^m_{kA}-\hat{X}^m_{kB})+\frac{(1-\alpha_d)\sigma_d^2}{\sigma_X^2+\sigma_d^2}\Delta\mu_X\right)F_\gamma\left(\frac{V_k\left(\hat{X}^m_{kA}-\hat{X}^m_{kB}\right)}{\sigma_p}\right)\right.$$
$$\left.\mathcal{I}\left\{-K^m<\hat{X}^m_{kA}-\hat{X}^m_{kB}<0\right\}\right]$$
$$= -\frac{\sigma_X^2+\alpha_d\sigma_d^2}{\sigma_X^2+\sigma_d^2}\mathbb{E}_{V_k,S_{kt}}\left[V_k\left(\hat{X}^m_{kA}-\hat{X}^m_{kB}\right)F_\gamma\left(\frac{V_k\left(\hat{X}^m_{kA}-\hat{X}^m_{kB}\right)}{\sigma_p}\right)\mathcal{I}\left\{-K^m<\hat{X}^m_{kA}-\hat{X}^m_{kB}<0\right\}\right]$$
$$-\frac{(1-\alpha_d)\sigma_d^2}{\sigma_X^2+\sigma_d^2}\Delta\mu_X\mathbb{E}_{V_k,S_{kt}}\left[V_kF_\gamma\left(\frac{V_k\left(\hat{X}^m_{kA}-\hat{X}^m_{kB}\right)}{\sigma_p}\right)\mathcal{I}\left\{-K^m<\hat{X}^m_{kA}-\hat{X}^m_{kB}<0\right\}\right]$$
$$= -\frac{\sigma_X^2+\alpha_d\sigma_d^2}{\sigma_X^2+\sigma_d^2}\mathbb{E}_{S_{kt}}\left[\mathbb{E}_{V_k}\left[V_k\left(\hat{X}^m_{kA}-\hat{X}^m_{kB}\right)F_\gamma\left(\frac{V_k\left(\hat{X}^m_{kA}-\hat{X}^m_{kB}\right)}{\sigma_p}\right)\mathcal{I}\left\{-K^m<\hat{X}^m_{kA}-\hat{X}^m_{kB}<0\right\}\middle|S_{kt}\right]\right]$$
$$-\frac{(1-\alpha_d)\sigma_d^2}{\sigma_X^2+\sigma_d^2}\Delta\mu_X\mathbb{E}_{S_{kt}}\left[\mathbb{E}_{V_k}\left[V_kF_\gamma\left(\frac{V_k\left(\hat{X}^m_{kA}-\hat{X}^m_{kB}\right)}{\sigma_p}\right)\mathcal{I}\left\{-K^m<\hat{X}^m_{kA}-\hat{X}^m_{kB}<0\right\}\middle|S_{kt}\right]\right]$$
$$= -\frac{\sigma_X^2+\alpha_d\sigma_d^2}{\sigma_X^2+\sigma_d^2}\mathbb{E}_{S_{kt}}\left[\mathbb{E}_{V_k}\left[V_k\left(\hat{X}^m_{kA}-\hat{X}^m_{kB}\right)F_\gamma\left(\frac{V_k\left(\hat{X}^m_{kA}-\hat{X}^m_{kB}\right)}{\sigma_p}\right)\mathcal{I}\left\{-K^m<\hat{X}^m_{kA}-\hat{X}^m_{kB}<0\right\}\right]\right]$$
$$-\frac{(1-\alpha_d)\sigma_d^2}{\sigma_X^2+\sigma_d^2}\Delta\mu_X\mathbb{E}_{S_{kt}}\left[\mathbb{E}_{V_k}\left[V_kF_\gamma\left(\frac{V_k\left(\hat{X}^m_{kA}-\hat{X}^m_{kB}\right)}{\sigma_p}\right)\mathcal{I}\left\{-K^m<\hat{X}^m_{kA}-\hat{X}^m_{kB}<0\right\}\right]\right]$$
$$\geq 0,$$

where $K^m=\alpha_p\sigma_pf(\sigma_V)$ (see Lemma 1), $P(V_k,\hat{x}_{kt})$ and $F_\gamma(.)$ are given with (13) and (28), respectively. The second equality follows from replacing $P\left(V_k,\hat{X}^m_{kt}\right)$ with the cdf of the standard logistic distribution, $F_\gamma(.)$, the third equality follows from (45) in Lemma 1, the forth equality follows from substituting the expression in (63) for $\mathbb{E}[X_{kA}\mid S_{kA}]-\mathbb{E}[X_{kB}\mid S_{kB}]$, the fifth equality follows from the linearity of expectation, the sixth equality follows from the law of iterated expectations, the seventh equality follows from the independence of V_k and S_{kt}, and the last inequality follows from (64), (65) and $\alpha_p,\alpha_d\leq 1$.

(b) In this part, we consider a miscalibrated doctor who is underconfident, i.e., $\alpha_p,\alpha_d>1$. First, note that using (11) and (22), $\mathbb{E}[X_{kA}\mid S_{kA}]-\mathbb{E}[X_{kB}\mid S_{kB}]$ could be written in terms of $\hat{X}^m_{kA}-\hat{X}^m_{kB}$ as follows:

$$\mathbb{E}[X_{kA} \mid S_{kA}] - \mathbb{E}[X_{kB} \mid S_{kB}] = (\hat{X}^m_{kA} - \hat{X}^m_{kB}) + \frac{(\alpha_d - 1)\sigma_X^2\sigma_d^2}{(\sigma_X^2 + \sigma_d^2)(\sigma_X^2 + \alpha_d\sigma_d^2)}(S_{kA} - S_{kB} - \Delta\mu_X). \tag{66}$$

Moreover, we can, alternatively, write $\hat{X}^m_{kA} - \hat{X}^m_{kB}$ as follows using (9) and (22).

$$\hat{X}^m_{kA} - \hat{X}^m_{kB} = \frac{\sigma_X^2}{\sigma_X^2 + \alpha_d\sigma_d^2}(S_{kA} - S_{kB}) + \frac{\alpha_d\sigma_d^2}{\sigma_X^2 + \alpha_d\sigma_d^2}\Delta\mu_X = \Delta\mu_X + \bar{\sigma}Z. \tag{67}$$

where Z is a standard normal random variable, and $\bar{\sigma}$ is given with (31). Finally, in the proof, we will need the following inequalities, which hold by (43) and (44) in Lemma 1 when we let $\alpha_p = 1$:

$$\mathbb{E}_{V_k}\left[V_k F_\gamma\left(\frac{V_k(\hat{x}^m_{kA} - \hat{x}^m_{kB})}{\sigma_p}\right)\right] < 0; \text{ for all } \hat{x}^m_{kt} \text{ with } \hat{x}^m_{kA} - \hat{x}^m_{kB} < -K, \text{ and} \tag{68}$$

$$\mathbb{E}_{V_k}\left[V_k(\hat{x}^m_{kA} - \hat{x}^m_{kB}) F_\gamma\left(\frac{V_k(\hat{x}^m_{kA} - \hat{x}^m_{kB})}{\sigma_p}\right)\right] \geq 0; \text{ for all } \hat{x}^m_{kt} \text{ with } \hat{x}^m_{kA} - \hat{x}^m_{kB} \geq 0 \text{ or } \hat{x}^m_{kA} - \hat{x}^m_{kB} \leq -K, \tag{69}$$

where $K = \sigma_p f(\sigma_V)$. Following the same steps as in (60) and replacing $\hat{X}_{kt}$ there with $\hat{X}^m_{kt}$, the expected utility difference between CbC and EBM for the miscalibrated doctor could be written as follows:

$$\begin{aligned}
U_d^{CE} &= \mathbb{E}_{V_k,S_{kt}}\left[V_k\left(\mathbb{E}[X_{kA} \mid S_{kA}] - \mathbb{E}[X_{kB} \mid S_{kB}]\right) P\left(V_k, \hat{X}^m_{kt}\right) \mathcal{I}\left\{U^{SDM}(\hat{X}^m_{kt}) - U^{EBM}(\hat{X}^m_{kt}) \geq 0\right\}\right] \\
&= \mathbb{E}_{V_k,S_{kt}}\left[V_k\left(\mathbb{E}[X_{kA} \mid S_{kA}] - \mathbb{E}[X_{kB} \mid S_{kB}]\right) F_\gamma\left(\frac{V_k\left(\hat{X}^m_{kA} - \hat{X}^m_{kB}\right)}{\sigma_p}\right)\right. \\
&\qquad\qquad \left.\mathcal{I}\left\{U^{SDM}(\hat{X}^m_{kt}) - U^{EBM}(\hat{X}^m_{kt}) \geq 0\right\}\right] \\
&= \mathbb{E}_{V_k,S_{kt}}\left[V_k\left(\mathbb{E}[X_{kA} \mid S_{kA}] - \mathbb{E}[X_{kB} \mid S_{kB}]\right) F_\gamma\left(\frac{V_k\left(\hat{X}^m_{kA} - \hat{X}^m_{kB}\right)}{\sigma_p}\right) \mathcal{I}\left\{\hat{X}^m_{kA} - \hat{X}^m_{kB} \geq 0\right\}\right] \\
&\quad + \mathbb{E}_{V_k,S_{kt}}\left[V_k\left(\mathbb{E}[X_{kA} \mid S_{kA}] - \mathbb{E}[X_{kB} \mid S_{kB}]\right) F_\gamma\left(\frac{V_k\left(\hat{X}^m_{kA} - \hat{X}^m_{kB}\right)}{\sigma_p}\right) \mathcal{I}\left\{\hat{X}^m_{kA} - \hat{X}^m_{kB} \leq -K^m\right\}\right] \\
&= \mathbb{E}_{V_k,S_{kt}}\left[V_k\left(\hat{X}^m_{kA} - \hat{X}^m_{kB} + \frac{(\alpha_d - 1)\sigma_X^2\sigma_d^2}{(\sigma_X^2 + \sigma_d^2)(\sigma_X^2 + \alpha_d\sigma_d^2)}(S_{kA} - S_{kB} - \Delta\mu_X)\right) F_\gamma\left(\frac{V_k\left(\hat{X}^m_{kA} - \hat{X}^m_{kB}\right)}{\sigma_p}\right)\right. \\
&\qquad\qquad \left.\mathcal{I}\left\{\hat{X}^m_{kA} - \hat{X}^m_{kB} \geq 0\right\}\right] \\
&\quad + \mathbb{E}_{V_k,S_{kt}}\left[V_k\left(\hat{X}^m_{kA} - \hat{X}^m_{kB} + \frac{(\alpha_d - 1)\sigma_X^2\sigma_d^2}{(\sigma_X^2 + \sigma_d^2)(\sigma_X^2 + \alpha_d\sigma_d^2)}(S_{kA} - S_{kB} - \Delta\mu_X)\right) F_\gamma\left(\frac{V_k\left(\hat{X}^m_{kA} - \hat{X}^m_{kB}\right)}{\sigma_p}\right)\right. \\
&\qquad\qquad \left.\mathcal{I}\left\{\hat{X}^m_{kA} - \hat{X}^m_{kB} \leq -K^m\right\}\right] \\
&= \mathbb{E}_{V_k,S_{kt}}\left[V_k\left(\hat{X}^m_{kA} - \hat{X}^m_{kB}\right) F_\gamma\left(\frac{V_k\left(\hat{X}^m_{kA} - \hat{X}^m_{kB}\right)}{\sigma_p}\right) \mathcal{I}\left\{\hat{X}^m_{kA} - \hat{X}^m_{kB} \geq 0\right\}\right]
\end{aligned}$$

$$
\begin{aligned}
&+\mathbb{E}_{V_k,S_{kt}}\left[V_k\left(\hat{X}^m_{kA}-\hat{X}^m_{kB}\right)F_\gamma\left(\frac{V_k\left(\hat{X}^m_{kA}-\hat{X}^m_{kB}\right)}{\sigma_p}\right)\mathcal{I}\left\{\hat{X}^m_{kA}-\hat{X}^m_{kB}\le -K^m\right\}\right]\\
&+\frac{\sigma_X^2(\alpha_d-1)\sigma_d^2}{(\sigma_X^2+\alpha_d\sigma_d^2)(\sigma_X^2+\sigma_d^2)}\mathbb{E}_{V_k,S_{kt}}\left[V_k(S_{kA}-S_{kB}-\Delta\mu_X)F_\gamma\left(\frac{V_k\left(\Delta\mu_X+\bar{\sigma}Z\right)}{\sigma_p}\right)\mathcal{I}\left\{\hat{X}^m_{kA}-\hat{X}^m_{kB}\ge 0\right\}\right]\\
&\quad+\frac{\sigma_X^2(\alpha_d-1)\sigma_d^2}{(\sigma_X^2+\alpha_d\sigma_d^2)(\sigma_X^2+\sigma_d^2)}\mathbb{E}_{V_k,S_{kt}}\left[V_k(S_{kA}-S_{kB}-\Delta\mu_X)F_\gamma\left(\frac{V_k\left(\Delta\mu_X+\bar{\sigma}Z\right)}{\sigma_p}\right)\mathcal{I}\left\{\hat{X}^m_{kA}-\hat{X}^m_{kB}\le -K^m\right\}\right]\\
&=\mathbb{E}_{V_k,S_{kt}}\left[V_k\left(\hat{X}^m_{kA}-\hat{X}^m_{kB}\right)F_\gamma\left(\frac{V_k\left(\hat{X}^m_{kA}-\hat{X}^m_{kB}\right)}{\sigma_p}\right)\mathcal{I}\left\{\hat{X}^m_{kA}-\hat{X}^m_{kB}\ge 0\right\}\right]\\
&\quad+\mathbb{E}_{V_k,S_{kt}}\left[V_k\left(\hat{X}^m_{kA}-\hat{X}^m_{kB}\right)F_\gamma\left(\frac{V_k\left(\hat{X}^m_{kA}-\hat{X}^m_{kB}\right)}{\sigma_p}\right)\mathcal{I}\left\{\hat{X}^m_{kA}-\hat{X}^m_{kB}\le -K^m\right\}\right]\\
&\quad+\frac{\sigma_X^2(\alpha_d-1)\sigma_d^2}{(\sigma_X^2+\alpha_d\sigma_d^2)\sqrt{\sigma_X^2+\sigma_d^2}}\mathbb{E}_{V_k,Z}\left[V_kZF_\gamma\left(\frac{V_k\left(\Delta\mu_X+\bar{\sigma}Z\right)}{\sigma_p}\right)\mathcal{I}\left\{\Delta\mu_X+\bar{\sigma}Z\ge 0\right\}\right]\\
&\quad\quad+\frac{\sigma_X^2(\alpha_d-1)\sigma_d^2}{(\sigma_X^2+\alpha_d\sigma_d^2)\sqrt{\sigma_X^2+\sigma_d^2}}\mathbb{E}_{V_k,Z}\left[V_kZF_\gamma\left(\frac{V_k\left(\Delta\mu_X+\bar{\sigma}Z\right)}{\sigma_p}\right)\mathcal{I}\left\{\Delta\mu_X+\bar{\sigma}Z\le -K^m\right\}\right]\\
&=\mathbb{E}_{V_k,S_{kt}}\left[V_k\left(\hat{X}^m_{kA}-\hat{X}^m_{kB}\right)F_\gamma\left(\frac{V_k\left(\hat{X}^m_{kA}-\hat{X}^m_{kB}\right)}{\sigma_p}\right)\mathcal{I}\left\{\hat{X}^m_{kA}-\hat{X}^m_{kB}\ge 0\right\}\right]\\
&\quad+\mathbb{E}_{V_k,S_{kt}}\left[V_k\left(\hat{X}^m_{kA}-\hat{X}^m_{kB}\right)F_\gamma\left(\frac{V_k\left(\hat{X}^m_{kA}-\hat{X}^m_{kB}\right)}{\sigma_p}\right)\mathcal{I}\left\{\hat{X}^m_{kA}-\hat{X}^m_{kB}\le -K^m\right\}\right]\\
&\quad\quad+\frac{\sigma_X^2(\alpha_d-1)\sigma_d^2}{(\sigma_X^2+\alpha_d\sigma_d^2)\sqrt{\sigma_X^2+\sigma_d^2}}\int_{\underline{v}}^{\bar{v}}\int_{\frac{-\Delta\mu_X}{\bar{\sigma}}}^{\infty}\frac{v_k}{\bar{v}-\underline{v}}zF_\gamma\left(\frac{v_k\left(\Delta\mu_X+\bar{\sigma}z\right)}{\sigma_p}\right)f_Z(z)dzdv_k\\
&\quad\quad\quad+\frac{\sigma_X^2(\alpha_d-1)\sigma_d^2}{(\sigma_X^2+\alpha_d\sigma_d^2)\sqrt{\sigma_X^2+\sigma_d^2}}\int_{\underline{v}}^{\bar{v}}\int_{-\infty}^{\frac{-K^m-\Delta\mu_X}{\bar{\sigma}}}\frac{v_k}{\bar{v}-\underline{v}}zF_\gamma\left(\frac{v_k\left(\Delta\mu_X+\bar{\sigma}z\right)}{\sigma_p}\right)f_Z(z)dzdv_k\\
&=\mathbb{E}_{V_k,S_{kt}}\left[V_k\left(\hat{X}^m_{kA}-\hat{X}^m_{kB}\right)F_\gamma\left(\frac{V_k\left(\hat{X}^m_{kA}-\hat{X}^m_{kB}\right)}{\sigma_p}\right)\mathcal{I}\left\{\hat{X}^m_{kA}-\hat{X}^m_{kB}\ge 0\right\}\right]\\
&\quad+\mathbb{E}_{V_k,S_{kt}}\left[V_k\left(\hat{X}^m_{kA}-\hat{X}^m_{kB}\right)F_\gamma\left(\frac{V_k\left(\hat{X}^m_{kA}-\hat{X}^m_{kB}\right)}{\sigma_p}\right)\mathcal{I}\left\{\hat{X}^m_{kA}-\hat{X}^m_{kB}\le -K^m\right\}\right]\\
&\quad\quad+\frac{\sigma_X^2(\alpha_d-1)\sigma_d^2}{(\sigma_X^2+\alpha_d\sigma_d^2)\sqrt{\sigma_X^2+\sigma_d^2}}\frac{\bar{\sigma}}{\sigma_p}\int_{\underline{v}}^{\bar{v}}\int_{\frac{-\Delta\mu_X}{\bar{\sigma}}}^{\infty}\frac{v_k^2}{\bar{v}-\underline{v}}f_\gamma\left(\frac{v_k\left(\Delta\mu_X+\bar{\sigma}z\right)}{\sigma_p}\right)f_Z(z)dzdv_k\\
&\quad\quad\quad+\frac{\sigma_X^2(\alpha_d-1)\sigma_d^2}{(\sigma_X^2+\alpha_d\sigma_d^2)\sqrt{\sigma_X^2+\sigma_d^2}}\mu_VF_\gamma(0)f_Z\left(\frac{-\Delta\mu_X}{\bar{\sigma}}\right)\\
&\quad\quad\quad\quad+\frac{\sigma_X^2(\alpha_d-1)\sigma_d^2}{(\sigma_X^2+\alpha_d\sigma_d^2)\sqrt{\sigma_X^2+\sigma_d^2}}\frac{\bar{\sigma}}{\sigma_p}\int_{\underline{v}}^{\bar{v}}\int_{-\infty}^{\frac{-K^m-\Delta\mu_X}{\bar{\sigma}}}\frac{v_k^2}{\bar{v}-\underline{v}}f_\gamma\left(\frac{v_k\left(\Delta\mu_X+\bar{\sigma}z\right)}{\sigma_p}\right)f_Z(z)dzdv_k\\
&\quad\quad\quad\quad\quad-\frac{\sigma_X^2(\alpha_d-1)\sigma_d^2}{(\sigma_X^2+\alpha_d\sigma_d^2)\sqrt{\sigma_X^2+\sigma_d^2}}\mathbb{E}_{V_k}\left[V_kF_\gamma\left(\frac{-K^m}{\sigma_p}\right)\right]f_Z\left(\frac{-K^m-\Delta\mu_X}{\bar{\sigma}}\right)
\end{aligned}
$$

$\geq 0,$

where $K^m = \alpha_p \sigma_p f(\sigma_V)$ (see Lemma 1), Z is a standard normal random variable, and $P(V_k, \hat{x}_{kt})$ and $F_\gamma(.)$ are given with (13) and (28), respectively. The second equality follows from replacing $P\left(V_k, \hat{X}^m_{kt}\right)$ with the cdf of the standard logistic distribution, $F_\gamma(.)$, the third equality follows from (44) in Lemma 1, the forth equality follows from substituting the expression in (66) for $\mathbb{E}[X_{kA} \mid S_{kA}] - \mathbb{E}[X_{kB} \mid S_{kB}]$, the fifth equality follows from the linearity of expectation, the sixth equality follows from (67) and replacing $S_{kA} - S_{kB} - \Delta\mu_X$ with $\sqrt{\sigma_X^2 + \sigma_d^2} Z$, the seventh equality follows from the definition of the expectation, and the eighth equality follows from integration by parts. Finally, the last inequality follows from (68), (69) and $\alpha_p, \alpha_d \geq 1$.

□

B.4. Proof of Section 7 Results

For the proof of Proposition 6, we need the following lemmas, which we prove in Appendix C.

Lemma A2 *Let Z_v and Z denote standard uniform and standard normal random variables, respectively. Note that Z_v is distributed between $-\sqrt{3}$ and $\sqrt{3}$, where $\underline{v} = \mu_V - \sqrt{3}\sigma_V$ and $\bar{v} = \mu_V + \sqrt{3}\sigma_V$. Furthermore, let G denote:*

$$G := \mathbb{E}_{Z_v, Z}\left[Z_v(\Delta\mu_X + \bar{\sigma}_1 Z) F_\gamma\left(\frac{(\mu_V + \sigma_V Z_v)(\Delta\mu_X + \bar{\sigma} Z)}{\sigma_p}\right)\mathcal{I}\{Z \in A\}\right] + \mathbb{E}_{Z_v, Z}\Big[\frac{Z_v}{\sigma_p}(\mu_V + \sigma_V Z_v)(\Delta\mu_X + \bar{\sigma}_1 Z)(\Delta\mu_X + \bar{\sigma} Z) f_\gamma\left(\frac{(\mu_V + \sigma_V Z_v)(\Delta\mu_X + \bar{\sigma} Z)}{\sigma_p}\right)\mathcal{I}\{Z \in A\}\Big], \tag{70}$$

where A is an integrable set, $\bar{\sigma}_1$ and $\bar{\sigma}$ are given with (30) and (31) with $0 \leq \alpha_d \leq 1$. If the condition

$$\mathbb{E}_Z\Big[(\Delta\mu_X + \bar{\sigma}_1 Z)(\Delta\mu_X + \bar{\sigma} Z) f_\gamma\Big(\frac{(\mu_V + \sigma_V Z_v)(\Delta\mu_X + \bar{\sigma} Z)}{\sigma_p}\Big)\mathcal{I}\{Z \in A\} \mid Z_v = z_v\Big] \geq 0 \tag{71}$$

holds for any $z_v \in [-\sqrt{3}, \sqrt{3}]$, we have $G \geq 0$.

Lemma A3 *Let Z denote a standard normal variable. The following inequality*

$$\mathbb{E}_Z\Big[(\Delta\mu_X + \bar{\sigma}_1 Z)(\Delta\mu_X + \bar{\sigma} Z) f_\gamma\Big(\frac{V_k(\Delta\mu_X + \bar{\sigma} Z)}{\sigma_p}\Big) \mid V_k = v_k\Big] \geq 0 \tag{72}$$

holds for any $v_k \in [\underline{v}, \bar{v}]$, where $\bar{\sigma}_1$ and $\bar{\sigma}$ are given with (30) and (31) with $0 \leq \alpha_d \leq 1$.

Now, we are ready to prove Proposition 6 using Lemmas A2 and A3. Note that throughout the proof of Proposition 6, we interchange the derivative and expectation by resting on the Dominated Convergence Theorem.

Proof of Proposition 6: We will first derive the expression for the expected utility difference between SDM and EBM when the doctor is miscalibrated. Following the same steps as in (58) and replacing $\hat{X}_{kt}$ with $\hat{X}^m_{kt}$ in (58), we can write the expected utility difference between SDM and EBM for the miscalibrated doctor as:

$$\begin{aligned} U_d^{SE} &= \mathbb{E}_{V_k,S_{kt}}\left[V_k\left(\mathbb{E}[X_{kA}\mid S_{kA}]-\mathbb{E}[X_{kB}\mid S_{kB}]\right)F_\gamma\left(\frac{V_k\left(\hat{X}^m_{kA}-\hat{X}^m_{kB}\right)}{\sigma_p}\right)\right] \\ &= \mathbb{E}_{V_k,Z}\left[V_k(\Delta\mu_X+\bar{\sigma}_1 Z)F_\gamma\left(\frac{V_k\left(\Delta\mu_X+\bar{\sigma}Z\right)}{\sigma_p}\right)\right], \end{aligned} \tag{73}$$

where the second equality follows from (59) and (67).

(a) Replacing V_k with $\mu_V+\sigma_V Z_v$ in (73), where Z_v is a standard uniform random variable distributed between $-\sqrt{3}$ and $\sqrt{3}$, U_d^{SE} could be written as:

$$U_d^{SE} = \mathbb{E}_{Z_v,Z}\Big[(\mu_V+\sigma_V Z_v)(\Delta\mu_X+\bar{\sigma}_1 Z)F_\gamma\Big(\frac{(\mu_V+\sigma_V Z_v)(\Delta\mu_X+\bar{\sigma}Z)}{\sigma_p}\Big)\Big]. \tag{74}$$

The derivative of U_d^{SE} in (74) with respect to σ_V is:

$$\begin{aligned} \frac{dU_d^{SE}}{d\sigma_V} &= \mathbb{E}_{Z_v,Z}\Big[Z_v(\Delta\mu_X+\bar{\sigma}_1 Z)F_\gamma\Big(\frac{(\mu_V+\sigma_V Z_v)(\Delta\mu_X+\bar{\sigma}Z)}{\sigma_p}\Big)\Big] \\ &\quad + \mathbb{E}_{Z_v,Z}\left[\frac{Z_v(\mu_V+\sigma_V Z_v)}{\sigma_p}(\Delta\mu_X+\bar{\sigma}_1 Z)(\Delta\mu_X+\bar{\sigma}Z)f_\gamma\Big(\frac{(\mu_V+\sigma_V Z_v)(\Delta\mu_X+\bar{\sigma}Z)}{\sigma_p}\Big)\right]. \end{aligned} \tag{75}$$

Case 1: $\alpha_d\in[0,1]$. When the doctor is not underconfident about their own errors, i.e., $\alpha_d\in[0,1]$, $\frac{dU_d^{SE}}{d\sigma_V}\geq 0$ immediately follows from Lemma A2 and Lemma A3.

Case 2: $\alpha_d\in(1,\infty)$. When the doctor is underconfident about their own errors, i.e., $\alpha_d\in(1,\infty)$, we rewrite $\frac{dU_d^{SE}}{d\sigma_V}$ in (75) as follows:

$$\begin{aligned} &\frac{dU_d^{SE}}{d\sigma_V} \\ &= \int_{-\sqrt{3}}^{\sqrt{3}}\int_{-\infty}^{\infty}\frac{z_v}{2\sqrt{3}}(\Delta\mu_X+\bar{\sigma}_1 z)F_\gamma\Big(\frac{(\mu_V+\sigma_V z_v)(\Delta\mu_X+\bar{\sigma}z)}{\sigma_p}\Big)f_Z(z)dzdz_v \\ &\quad+\int_{-\sqrt{3}}^{\sqrt{3}}\int_{-\infty}^{\infty}\frac{z_v(\mu_V+\sigma_V z_v)}{2\sqrt{3}\sigma_p}(\Delta\mu_X+\bar{\sigma}_1 z)(\Delta\mu_X+\bar{\sigma}z)f_\gamma\Big(\frac{(\mu_V+\sigma_V z_v)(\Delta\mu_X+\bar{\sigma}z)}{\sigma_p}\Big)f_Z(z)dzdz_v \\ &= \int_{-\sqrt{3}}^{\sqrt{3}}\int_{-\infty}^{\infty}\frac{z_v}{2\sqrt{3}\bar{\sigma}}(\Delta\mu_X+\bar{\sigma}z)((\Delta\mu_X+\bar{\sigma}_1 z)z-\bar{\sigma}_1)F_\gamma\Big(\frac{(\mu_V+\sigma_V z_v)(\Delta\mu_X+\bar{\sigma}z)}{\sigma_p}\Big)f_Z(z)dzdz_v, \end{aligned} \tag{76}$$

where the first equality follows from the definition of expectation, and the last equality follows from integration by parts. Now, let U_1 and U_2 denote:

$$\begin{aligned} U_1 = &-\int_{-\sqrt{3}}^{\sqrt{3}}\int_{-\infty}^{\infty}\frac{\bar{\sigma}_1 z_v}{2\sqrt{3}\bar{\sigma}}(\Delta\mu_X+\bar{\sigma}z)F_\gamma\Big(\frac{(\mu_V+\sigma_V z_v)(\Delta\mu_X+\bar{\sigma}z)}{\sigma_p}\Big)f_Z(z)dzdz_v \\ &+\int_{-\sqrt{3}}^{\sqrt{3}}\int_{0}^{-\Delta\mu_X/\bar{\sigma}_1}\frac{z_v}{2\sqrt{3}\bar{\sigma}}(\Delta\mu_X+\bar{\sigma}z)(\Delta\mu_X+\bar{\sigma}_1 z)zF_\gamma\Big(\frac{(\mu_V+\sigma_V z_v)(\Delta\mu_X+\bar{\sigma}z)}{\sigma_p}\Big)f_Z(z)dzdz_v, \end{aligned}$$

$$U_2 = \int_{-\sqrt{3}}^{\sqrt{3}}\int_{-\infty}^{0} \frac{z_v}{2\sqrt{3}\bar{\sigma}}(\Delta\mu_X + \bar{\sigma}z)(\Delta\mu_X + \bar{\sigma}_1 z)zF_\gamma\Big(\frac{(\mu_V + \sigma_V z_v)(\Delta\mu_X + \bar{\sigma}z)}{\sigma_p}\Big) f_Z(z)dzdz_v$$
$$+ \int_{-\sqrt{3}}^{\sqrt{3}}\int_{-\Delta\mu_X/\bar{\sigma}_1}^{\infty} \frac{z_v}{2\sqrt{3}\bar{\sigma}}(\Delta\mu_X + \bar{\sigma}z)(\Delta\mu_X + \bar{\sigma}_1 z)zF_\gamma\Big(\frac{(\mu_V + \sigma_V z_v)(\Delta\mu_X + \bar{\sigma}z)}{\sigma_p}\Big) f_Z(z)dzdz_v.$$

Then, $\frac{dU_d^{SE}}{d\sigma_V}$ in (76) is equal to $U_1 + U_2$. Furthermore, one can easily verify that U_1 decreases in σ_V, whereas U_2 increases in σ_V. Hence, we have the following inequalities for $\frac{dU_d^{SE}}{d\sigma_V}$:

$$\begin{aligned}\frac{dU_d^{SE}}{d\sigma_V} &\geq \Big(\lim_{\sigma_V\to\infty} U_1\Big) + U_2 \\ &= \frac{3}{4\sqrt{3}\bar{\sigma}}\Big(\Delta\mu_X\bar{\sigma}_1\mathbb{P}_Z(Z \leq -\Delta\mu_X/\bar{\sigma}) - \Delta\mu_X(\bar{\sigma}_1+\bar{\sigma})\left(\mathbb{P}_Z(Z\leq -\Delta\mu_X/\bar{\sigma}_1) - \mathbb{P}_Z(Z\leq 0)\right) \\ &\quad - \Delta\mu_X\bar{\sigma}_1\mathbb{P}_Z(Z\geq -\Delta\mu_X/\bar{\sigma}) - 2\bar{\sigma}\bar{\sigma}_1 f_Z(-\Delta\mu_X/\bar{\sigma}) - (\Delta\mu_X)^2 f_Z(0) \\ &\quad + 2\bar{\sigma}\bar{\sigma}_1 f_Z(-\Delta\mu_X/\bar{\sigma}_1) - 2\bar{\sigma}\bar{\sigma}_1 f_Z(0)\Big) + U_2.\end{aligned} \tag{77}$$

Since U_2 increases in σ_V, the expression in (77) is an increasing function of σ_V and it approaches $\frac{3}{4\sqrt{3}\bar{\sigma}}\Big(2\bar{\sigma}\bar{\sigma}_1 f_Z(-\Delta\mu_X/\bar{\sigma}) - \Delta\mu_X\bar{\sigma}(2\mathbb{P}_Z(Z\leq -\Delta\mu_X/\bar{\sigma}) - 1)\Big)$ as σ_V tends to infinity. This implies that there is a unique σ_V^* such that the expression in (77) is nonnegative if and only if $\sigma_V \geq \sigma_V^*$ holds, i.e., if σ_V is sufficiently high. Since the expression in (77) is a lower bound for $\frac{dU_d^{SE}}{d\sigma_V}$, we can conclude that $\frac{dU_d^{SE}}{d\sigma_V}$ is also nonnegative for sufficiently high σ_V ($\sigma_V \geq \sigma_V^*$).

(b) The derivative of U_d^{SE} with respect to σ_X^2 is:

$$\begin{aligned}\frac{dU_d^{SE}}{d(\sigma_X^2)} &= \frac{d\bar{\sigma}_1}{d(\sigma_X^2)}\mathbb{E}_{V_k,Z}\Big[V_k Z F_\gamma\Big(\frac{V_k(\Delta\mu_X+\bar{\sigma}Z)}{\sigma_p}\Big)\Big] + \frac{d\bar{\sigma}}{d(\sigma_X^2)}\mathbb{E}_{V_k,Z}\Big[\frac{V_k^2}{\sigma_p}Z(\Delta\mu_X+\bar{\sigma}_1 Z)f_\gamma\Big(\frac{V_k(\Delta\mu_X+\bar{\sigma}Z)}{\sigma_p}\Big)\Big] \\ &= \frac{d\bar{\sigma}_1}{d(\sigma_X^2)}\bar{\sigma}\mathbb{E}_{V_k,Z}\Big[\frac{V_k^2}{\sigma_p}f_\gamma\Big(\frac{V_k(\Delta\mu_X+\bar{\sigma}Z)}{\sigma_p}\Big)\Big] + \frac{d\bar{\sigma}}{d(\sigma_X^2)}\mathbb{E}_{V_k,Z}\Big[\frac{V_k^2}{\sigma_p}Z(\Delta\mu_X+\bar{\sigma}_1 Z)f_\gamma\Big(\frac{V_k(\Delta\mu_X+\bar{\sigma}Z)}{\sigma_p}\Big)\Big] \\ &= \frac{d\bar{\sigma}}{d(\sigma_X^2)}\bar{\sigma}_1\mathbb{E}_{V_k,Z}\Big[\frac{V_k^2}{\sigma_p}\left(Z^2 + \frac{\Delta\mu_X}{\bar{\sigma}_1}Z + \frac{\frac{d\bar{\sigma}_1}{d(\sigma_X^2)}\frac{1}{\bar{\sigma}_1}}{\frac{d\bar{\sigma}}{d(\sigma_X^2)}\frac{1}{\bar{\sigma}}}\right)f_\gamma\Big(\frac{V_k(\Delta\mu_X+\bar{\sigma}Z)}{\sigma_p}\Big)\Big],\end{aligned}$$

where it could be easily confirmed that $\frac{d\bar{\sigma}}{d(\sigma_X^2)} \geq 0$. Note that $z^2 + \frac{\Delta\mu_X}{\bar{\sigma}_1}z + \frac{\frac{d\bar{\sigma}_1}{d(\sigma_X^2)}\frac{1}{\bar{\sigma}_1}}{\frac{d\bar{\sigma}}{d(\sigma_X^2)}\frac{1}{\bar{\sigma}}} \geq 0$ holds for all $z \in (-\infty, \infty)$ if

$$\left(\frac{\Delta\mu_X}{\bar{\sigma}_1}\right)^2 \leq 4\frac{\frac{d\bar{\sigma}_1}{d(\sigma_X^2)}\frac{1}{\bar{\sigma}_1}}{\frac{d\bar{\sigma}}{d(\sigma_X^2)}\frac{1}{\bar{\sigma}}} \;\Leftrightarrow\; \Delta\mu_X^2 \leq 4\bar{\sigma}\bar{\sigma}_1\frac{\frac{d\bar{\sigma}_1}{d(\sigma_X^2)}}{\frac{d\bar{\sigma}}{d(\sigma_X^2)}}, \tag{78}$$

which ensures the nonnegativity of $\frac{dU_d^{SE}}{d(\sigma_X^2)}$. Since the right-hand side of (78) increases in σ_X, it tends to zero as σ_X goes to 0 and it tends to infinity as σ_X goes to infinity, we can conclude that there exists a unique σ_X^* that satisfies (78) with equality such that (78) holds if and only if $\sigma_X \geq \sigma_X^*$. This implies that $\frac{dU_d^{SE}}{d(\sigma_X^2)} \geq 0$ for $\sigma_X \geq \sigma_X^*$.

(c) The derivative of U_d^{SE} in (73) with respect to σ_p is:

$$\frac{dU_d^{SE}}{d\sigma_p} = -\mathbb{E}_{V_k,Z}\Big[\frac{V_k^2}{\sigma_p^2}(\Delta\mu_X+\bar{\sigma}_1 Z)(\Delta\mu_X+\bar{\sigma}Z)f_\gamma\Big(\frac{V_k(\Delta\mu_X+\bar{\sigma}Z)}{\sigma_p}\Big)\Big]. \tag{79}$$

Case 1: $\alpha_d\in[0,1]$. When the doctor is not underconfident about their own errors, i.e., $\alpha_d\in[0,1]$, $\frac{dU_d^{SE}}{d\sigma_p}\le 0$ immediately follows from Lemma A3.

Case 2: $\alpha_d\in(1,\infty)$. When the doctor is underconfident about their own errors, i.e., $\alpha_d\in(1,\infty)$, we rewrite $\frac{dU_d^{SE}}{d\sigma_p}$ in (79) as follows:

$$\begin{aligned}\frac{dU_d^{SE}}{d\sigma_p} &= -\int_{\underline{v}}^{\bar{v}}\int_{-\infty}^{\infty}\frac{v_k^2}{\sigma_p^2(\bar{v}-\underline{v})}(\Delta\mu_X+\bar{\sigma}_1 z)(\Delta\mu_X+\bar{\sigma}z)f_\gamma\Big(\frac{V_k(\Delta\mu_X+\bar{\sigma}z)}{\sigma_p}\Big)f_Z(z)dzdv_k\\
&= -\int_{\underline{v}}^{\bar{v}}\int_{-\Delta\mu_X/\bar{\sigma}_1}^{-\Delta\mu_X/\bar{\sigma}}\frac{v_k^2}{\sigma_p^2(\bar{v}-\underline{v})}(\Delta\mu_X+\bar{\sigma}_1 z)(\Delta\mu_X+\bar{\sigma}z)f_\gamma\Big(\frac{V_k(\Delta\mu_X+\bar{\sigma}z)}{\sigma_p}\Big)f_Z(z)dzdv_k\\
&\quad -\int_{\underline{v}}^{\bar{v}}\int_{-\infty}^{-\Delta\mu_X/\bar{\sigma}_1}\frac{v_k^2}{\sigma_p^2(\bar{v}-\underline{v})}(\Delta\mu_X+\bar{\sigma}_1 z)(\Delta\mu_X+\bar{\sigma}z)f_\gamma\Big(\frac{V_k(\Delta\mu_X+\bar{\sigma}z)}{\sigma_p}\Big)f_Z(z)dzdv_k\\
&\quad -\int_{\underline{v}}^{\bar{v}}\int_{-\Delta\mu_X/\bar{\sigma}}^{\infty}\frac{v_k^2}{\sigma_p^2(\bar{v}-\underline{v})}(\Delta\mu_X+\bar{\sigma}_1 z)(\Delta\mu_X+\bar{\sigma}z)f_\gamma\Big(\frac{V_k(\Delta\mu_X+\bar{\sigma}z)}{\sigma_p}\Big)f_Z(z)dzdv_k.\end{aligned}$$

Since we are interested only in the sign of $\frac{dU_d^{SE}}{d\sigma_p}$, we will focus on $\sigma_p^2\frac{dU_d^{SE}}{d\sigma_p}$, which has the same sign with $\frac{dU_d^{SE}}{d\sigma_p}$ but is easier to analyze:

$$\begin{aligned}\sigma_p^2\frac{dU_d^{SE}}{d\sigma_p} &= -\int_{\underline{v}}^{\bar{v}}\int_{-\Delta\mu_X/\bar{\sigma}_1}^{-\Delta\mu_X/\bar{\sigma}}\frac{v_k^2}{(\bar{v}-\underline{v})}(\Delta\mu_X+\bar{\sigma}_1 z)(\Delta\mu_X+\bar{\sigma}z)f_\gamma\Big(\frac{V_k(\Delta\mu_X+\bar{\sigma}z)}{\sigma_p}\Big)f_Z(z)dzdv_k\\
&\quad -\int_{\underline{v}}^{\bar{v}}\int_{-\infty}^{-\Delta\mu_X/\bar{\sigma}_1}\frac{v_k^2}{(\bar{v}-\underline{v})}(\Delta\mu_X+\bar{\sigma}_1 z)(\Delta\mu_X+\bar{\sigma}z)f_\gamma\Big(\frac{V_k(\Delta\mu_X+\bar{\sigma}z)}{\sigma_p}\Big)f_Z(z)dzdv_k\\
&\quad -\int_{\underline{v}}^{\bar{v}}\int_{-\Delta\mu_X/\bar{\sigma}}^{\infty}\frac{v_k^2}{(\bar{v}-\underline{v})}(\Delta\mu_X+\bar{\sigma}_1 z)(\Delta\mu_X+\bar{\sigma}z)f_\gamma\Big(\frac{V_k(\Delta\mu_X+\bar{\sigma}z)}{\sigma_p}\Big)f_Z(z)dzdv_k.\end{aligned} \tag{80}$$

In (80), one can easily confirm that the first component increases in σ_p, whereas the second and third components decrease in σ_p. Hence, we have the following inequalities for $\sigma_p^2\frac{dU_d^{SE}}{d\sigma_p}$:

$$\begin{aligned}\sigma_p^2\frac{dU_d^{SE}}{d\sigma_p} &\le -\Big(\lim_{\sigma_p\to\infty}\int_{\underline{v}}^{\bar{v}}\int_{-\Delta\mu_X/\bar{\sigma}_1}^{-\Delta\mu_X/\bar{\sigma}}\frac{v_k^2}{(\bar{v}-\underline{v})}(\Delta\mu_X+\bar{\sigma}_1 z)(\Delta\mu_X+\bar{\sigma}z)f_\gamma\Big(\frac{V_k(\Delta\mu_X+\bar{\sigma}z)}{\sigma_p}\Big)f_Z(z)dzdv_k\Big)\\
&\quad -\int_{\underline{v}}^{\bar{v}}\int_{-\infty}^{-\Delta\mu_X/\bar{\sigma}_1}\frac{v_k^2}{(\bar{v}-\underline{v})}(\Delta\mu_X+\bar{\sigma}_1 z)(\Delta\mu_X+\bar{\sigma}z)f_\gamma\Big(\frac{V_k(\Delta\mu_X+\bar{\sigma}z)}{\sigma_p}\Big)f_Z(z)dzdv_k\\
&\quad -\int_{\underline{v}}^{\bar{v}}\int_{-\Delta\mu_X/\bar{\sigma}}^{\infty}\frac{v_k^2}{(\bar{v}-\underline{v})}(\Delta\mu_X+\bar{\sigma}_1 z)(\Delta\mu_X+\bar{\sigma}z)f_\gamma\Big(\frac{V_k(\Delta\mu_X+\bar{\sigma}z)}{\sigma_p}\Big)f_Z(z)dzdv_k\\
&= -\mathbb{E}[V_k^2]f_\gamma(0)\Big((\Delta\mu_X^2+\bar{\sigma}\bar{\sigma}_1)\mathbb{P}_Z(-\Delta\mu_X/\bar{\sigma}_1\le Z\le -\Delta\mu_X/\bar{\sigma})\\
&\qquad\qquad +\Delta\mu_X\bar{\sigma}_1 f_Z(-\Delta\mu_X/\bar{\sigma})-\Delta\mu_X\bar{\sigma}f_Z(-\Delta\mu_X/\bar{\sigma}_1)\Big)\\
&\quad -\int_{\underline{v}}^{\bar{v}}\int_{-\infty}^{-\Delta\mu_X/\bar{\sigma}_1}\frac{v_k^2}{(\bar{v}-\underline{v})}(\Delta\mu_X+\bar{\sigma}_1 z)(\Delta\mu_X+\bar{\sigma}z)f_\gamma\Big(\frac{V_k(\Delta\mu_X+\bar{\sigma}z)}{\sigma_p}\Big)f_Z(z)dzdv_k\end{aligned}$$

$$-\int_{\underline{v}}^{\bar{v}}\int_{-\Delta\mu_X/\bar{\sigma}}^{\infty}\frac{v_k^2}{(\bar{v}-\underline{v})}(\Delta\mu_X+\bar{\sigma}_1 z)(\Delta\mu_X+\bar{\sigma}z)f_\gamma\Big(\frac{V_k(\Delta\mu_X+\bar{\sigma}z)}{\sigma_p}\Big)f_Z(z)dzdv_k \tag{81}$$

Note that the expression in (81) is a decreasing function of σ_p, and it approaches $-\mathbb{E}[V_k^2]f_\gamma(0)(\Delta\mu_X^2+\bar{\sigma}\bar{\sigma}_1)$ as σ_p tends to infinity. This implies that there is a unique σ_p^* such that the expression in (80) is nonpositive if and only if $\sigma_p\geq\sigma_p^*$ holds, i.e., if σ_p is sufficiently high. Since the expression in (81) is an upper bound for $\sigma_p^2\frac{dU_d^{SE}}{d\sigma_p}$, we can conclude that $\sigma_p^2\frac{dU_d^{SE}}{d\sigma_p}$ is also nonpositive for sufficiently high σ_p ($\sigma_p\geq\sigma_p^*$).

(d) The derivative of U_d^{SE} with respect to σ_d^2 is:

$$\begin{aligned}\frac{dU_d^{SE}}{d(\sigma_d^2)}&=\frac{d\bar{\sigma}_1}{d(\sigma_d^2)}\mathbb{E}_{V_k,Z}\Big[V_kZF_\gamma\Big(\frac{V_k(\Delta\mu_X+\bar{\sigma}Z)}{\sigma_p}\Big)\Big]+\frac{d\bar{\sigma}}{d(\sigma_d^2)}\mathbb{E}_{V_k,Z}\Big[\frac{V_k^2}{\sigma_p}Z(\Delta\mu_X+\bar{\sigma}_1Z)f_\gamma\Big(\frac{V_k(\Delta\mu_X+\bar{\sigma}Z)}{\sigma_p}\Big)\Big]\\&=\frac{d\bar{\sigma}_1}{d(\sigma_d^2)}\bar{\sigma}\mathbb{E}_{V_k,Z}\Big[\frac{V_k^2}{\sigma_p}f_\gamma\Big(\frac{V_k(\Delta\mu_X+\bar{\sigma}Z)}{\sigma_p}\Big)\Big]+\frac{d\bar{\sigma}}{d(\sigma_d^2)}\mathbb{E}_{V_k,Z}\Big[\frac{V_k^2}{\sigma_p}Z(\Delta\mu_X+\bar{\sigma}_1Z)f_\gamma\Big(\frac{V_k(\Delta\mu_X+\bar{\sigma}Z)}{\sigma_p}\Big)\Big]\\&=\frac{d\bar{\sigma}}{d(\sigma_d^2)}\bar{\sigma}_1\mathbb{E}_{V_k,Z}\Big[\frac{V_k^2}{\sigma_p}\left(Z^2+\frac{\Delta\mu_X}{\bar{\sigma}_1}Z+\frac{\frac{d\bar{\sigma}_1}{d(\sigma_d^2)}\frac{1}{\bar{\sigma}_1}}{\frac{d\bar{\sigma}}{d(\sigma_d^2)}\frac{1}{\bar{\sigma}}}\right)f_\gamma\Big(\frac{V_k(\Delta\mu_X+\bar{\sigma}Z)}{\sigma_p}\Big)\Big],\end{aligned} \tag{82}$$

where $\frac{d\bar{\sigma}}{d(\sigma_d^2)}$ is equal to

$$\frac{d\bar{\sigma}}{d(\sigma_d^2)}=\frac{\sigma_X^2((1-2\alpha_d)\sigma_X^2-\alpha_d\sigma_d^2)}{2(\sigma_X^2+\alpha_d\sigma_d^2)^2\sqrt{\sigma_X^2+\sigma_d^2}}.$$

Note that $\frac{d\bar{\sigma}}{d(\sigma_d^2)}$ is nonpositive when $\alpha_d\geq 1/2$. Furthermore, $z^2+\frac{\Delta\mu_X}{\bar{\sigma}_1}z+\frac{\frac{d\bar{\sigma}_1}{d(\sigma_d^2)}\frac{1}{\bar{\sigma}_1}}{\frac{d\bar{\sigma}}{d(\sigma_d^2)}\frac{1}{\bar{\sigma}}}\geq 0$ follows for all $z\in(-\infty,\infty)$ if

$$\left(\frac{\Delta\mu_X}{\bar{\sigma}_1}\right)^2\leq 4\frac{\frac{d\bar{\sigma}_1}{d(\sigma_d^2)}\frac{1}{\bar{\sigma}_1}}{\frac{d\bar{\sigma}}{d(\sigma_d^2)}\frac{1}{\bar{\sigma}}}\;\Leftrightarrow\;\Delta\mu_X^2\leq 4\bar{\sigma}\bar{\sigma}_1\frac{\frac{d\bar{\sigma}_1}{d(\sigma_d^2)}}{\frac{d\bar{\sigma}}{d(\sigma_d^2)}}, \tag{83}$$

and this ensures the nonpositivity of $\frac{dU_d^{SE}}{d(\sigma_d^2)}$. The right-hand side of (83) decreases in σ_d (for $\alpha_d\in[1/2,\infty)$), it tends to zero as σ_d goes to infinity and it tends to $\frac{\sigma_X^2}{2\alpha_d-1}$ as σ_d goes to zero. Hence, if $\frac{\sigma_X^2}{2\alpha_d-1}\geq\Delta\mu_X^2\Leftrightarrow\sigma_X\geq-\Delta\mu_X\sqrt{2\alpha_d-1}$ holds, we can conclude that there exists a unique σ_d^* that satisfies (83) with equality such that (83) holds if and only if $\sigma_d\leq\sigma_d^*$. This implies that we have $\frac{dU_d^{SE}}{d(\sigma_d^2)}\leq 0$ for $\sigma_d\leq\sigma_d^*$ given that $\alpha_d\geq 1/2$ and $\sigma_X\geq-(\mu_A-\mu_B)\sqrt{2\alpha_d-1}$ hold.

□

Appendix C: Proofs of Appendix Lemmas

Proof of Lemma A1:

1. It is trivial to show that $H(y)=\mathbb{E}_{V_k}\left[V_kF_\gamma\left(\frac{V_ky}{\alpha_p\sigma_p}\right)\right]$ increases in y, it is strictly negative as y tends to infinity, and it is strictly positive when $y=0$. This implies that there exists a unique $y^*<0$ such that

$$\mathbb{E}_{V_k}\left[V_kF_\gamma\left(\frac{V_ky^*}{\alpha_p\sigma_p}\right)\right]=0, \tag{84}$$

$\mathbb{E}_{V_k}\left[V_kF_\gamma\left(\frac{V_ky}{\alpha_p\sigma_p}\right)\right]<0$ if $y<y^*$, and $\mathbb{E}_{V_k}\left[V_kF_\gamma\left(\frac{V_ky}{\alpha_p\sigma_p}\right)\right]>0$ if $y>y^*$.

2. Recall that y^* satisfies (84) when $y<0$. Applying implicit differentiation to (84), the derivative of y^* with respect to σ_p is:

$$\frac{dy^*}{d\sigma_p}=-\frac{\partial\mathbb{E}_{V_k}\left[V_kF_\gamma\left(\frac{V_ky}{\alpha_p\sigma_p}\right)\right]/\partial\sigma_p}{\partial\mathbb{E}_{V_k}\left[V_kF_\gamma\left(\frac{V_ky}{\alpha_p\sigma_p}\right)\right]/\partial y}=-\frac{-\frac{y}{\alpha_p\sigma_p^2}\mathbb{E}_{V_k}\left[V_k^2f_\gamma\left(\frac{V_ky}{\alpha_p\sigma_p}\right)\right]}{\frac{1}{\alpha_p\sigma_p}\mathbb{E}_{V_k}\left[V_k^2f_\gamma\left(\frac{V_ky}{\alpha_p\sigma_p}\right)\right]}=\frac{y}{\sigma_p},$$

where we interchange the expectation and the differentiation by resting on the Dominated Convergence Theorem, and $f_\gamma(.)$ is given with (29). The above equation implies that y^* is decreasing and linear in σ_p. We can show that y^* is decreasing and linear in α_p as well following the same steps as the proof for σ_p, and thus we skip it.

On the other hand, to analyze the behavior of y^* in σ_V, first let us rewrite (84) in the following way:

$$\mathbb{E}_{Z_v}\left[(\mu_V+\sigma_VZ_v)F_\gamma\left(\frac{(\mu_V+\sigma_VZ_v)y}{\alpha_p\sigma_p}\right)\right]=0, \tag{85}$$

where Z_v is a standard uniform random variable with mean zero and standard deviation 1, distributed between $-\sqrt{3}$ and $\sqrt{3}$. Furthermore, $\underline{v}$ in (84) is equal to $\mu_V-\sqrt{3}\sigma_V$ and $\bar{v}$ in (84) is equal to $\mu_V+\sqrt{3}\sigma_V$. Applying implicit differentiation to (85), the derivative of y^* with respect to σ_V is:

$$\begin{aligned}\frac{dy^*}{d\sigma_V}&=-\frac{\partial\mathbb{E}_{Z_v}\left[(\mu_V+\sigma_VZ_v)F_\gamma\left(\frac{(\mu_V+\sigma_VZ_v)y}{\sigma_p}\right)\right]/\partial\sigma_V}{\partial\mathbb{E}_{Z_v}\left[(\mu_V+\sigma_VZ_v)F_\gamma\left(\frac{(\mu_V+\sigma_VZ_v)y}{\sigma_p}\right)\right]/\partial y}\\&=-\frac{\mathbb{E}_{Z_v}\left[Z_vF_\gamma\left(\frac{(\mu_V+\sigma_VZ_v)y}{\alpha_p\sigma_p}\right)\right]+\frac{y}{\alpha_p\sigma_p}\mathbb{E}_{Z_v}\left[Z_v(\mu_V+\sigma_VZ_v)f_\gamma\left(\frac{(\mu_V+\sigma_VZ_v)y}{\alpha_p\sigma_p}\right)\right]}{\frac{1}{\alpha_p\sigma_p}\mathbb{E}_{Z_v}\left[(\mu_V+\sigma_VZ_v)^2f_\gamma\left(\frac{(\mu_V+\sigma_VZ_v)y}{\alpha_p\sigma_p}\right)\right]},\end{aligned}$$

where we interchange the expectation and the differentiation by resting on the Dominated Convergence Theorem. The denominator of the above expression is trivially nonnegative. The nonnegativity of the numerator follows from Claim 1 proved at the end of this lemma. This proves the increasing nature of y^* in σ_V. Finally, note that as $\sigma_V\to\mu_V/\sqrt{3}$ (i.e., $\underline{v}\to 0$ by (32)), $H(y)$ becomes nonnegative for all $y\in(-\infty,\infty)$, which implies that y^* tends to minus infinity and $f(\sigma_V)$ tends to plus infinity. On the other hand, as $\sigma_V\to\infty$, $H(y)$ is nonnegative for all $y\in[0,\infty)$, whereas it is negative for all $y\in(-\infty,0)$, which implies that both y^* and $f(\sigma_V)$ tend to zero.

Claim 1: Let G denote

$$G:=\mathbb{E}_{Z_v}\left[Z_vF_\gamma\left(\frac{(\mu_V+\sigma_VZ_v)y}{\alpha_p\sigma_p}\right)\right]+\frac{y}{\alpha_p\sigma_p}\mathbb{E}_{Z_v}\left[Z_v(\mu_V+\sigma_VZ_v)f_\gamma\left(\frac{(\mu_V+\sigma_VZ_v)y}{\alpha_p\sigma_p}\right)\right].$$

Furthermore, let $y<0$ and (85) hold. Then, we have $G\le 0$.

Proof of Claim 1: By the definition of expectation, G is equal to:

$$G=\int_{-\sqrt{3}}^{\sqrt{3}}z_vF_\gamma\left(\frac{(\mu_V+\sigma_Vz_v)y}{\alpha_p\sigma_p}\right)\frac{1}{2\sqrt{3}}dz_v+\frac{y}{\alpha_p\sigma_p}\int_{-\sqrt{3}}^{\sqrt{3}}z_v(\mu_V+\sigma_Vz_v)f_\gamma\left(\frac{(\mu_V+\sigma_Vz_v)y}{\alpha_p\sigma_p}\right)\frac{1}{2\sqrt{3}}dz_v. \tag{86}$$

Applying integration by parts to (86), we have:

$$\begin{aligned} G &= \int_{-\sqrt{3}}^{\sqrt{3}} z_v F_\gamma\left(\frac{(\mu_V+\sigma_V z_v)y}{\alpha_p\sigma_p}\right)\frac{1}{2\sqrt{3}}dz_v + \left[\frac{z_v(\mu_V+\sigma_V z_v)}{2\sqrt{3}\sigma_V}F_\gamma\left(\frac{(\mu_V+\sigma_V z_v)y}{\alpha_p\sigma_p}\right)\right]_{z_v=-\sqrt{3}}^{z_v=\sqrt{3}} \\ &\quad - \int_{-\sqrt{3}}^{\sqrt{3}} z_v F_\gamma\left(\frac{(\mu_V+\sigma_V z_v)y}{\alpha_p\sigma_p}\right)\frac{1}{2\sqrt{3}}dz_v - \int_{-\sqrt{3}}^{\sqrt{3}} \frac{\mu_V+\sigma_V z_v}{\sigma_V}F_\gamma\left(\frac{(\mu_V+\sigma_V z_v)y}{\alpha_p\sigma_p}\right)\frac{1}{2\sqrt{3}}dz_v \\ &= \frac{\bar{v}}{2\sigma_V}F_\gamma\left(\frac{\bar{v}y}{\alpha_p\sigma_p}\right) + \frac{\underline{v}}{2\sigma_V}F_\gamma\left(\frac{\underline{v}y}{\alpha_p\sigma_p}\right), \end{aligned} \tag{87}$$

where the second equality follows from (85). Applying integration by parts to (84) (or equivalently (85)), we have:

$$\frac{\bar{v}^2}{2(\bar{v}-\underline{v})}F_\gamma\left(\frac{\bar{v}y}{\alpha_p\sigma_p}\right) - \frac{\underline{v}^2}{2(\bar{v}-\underline{v})}F_\gamma\left(\frac{\underline{v}y}{\alpha_p\sigma_p}\right) - \frac{y}{\alpha_p\sigma_p}\int_{\underline{v}}^{\bar{v}}\frac{v_k^2}{2}f_\gamma\left(\frac{\bar{v}y}{\alpha_p\sigma_p}\right)\frac{1}{(\bar{v}-\underline{v})}dv_k = 0,$$

which implies from $y<0$ that

$$\bar{v}^2F_\gamma\left(\frac{\bar{v}y}{\alpha_p\sigma_p}\right) - \underline{v}^2F_\gamma\left(\frac{\underline{v}y}{\alpha_p\sigma_p}\right) \le 0. \tag{88}$$

Using (88) in (87), we obtain below inequality for G:

$$G \le \frac{\underline{v}^2}{2\sigma_V\bar{v}}F_\gamma\left(\frac{\underline{v}y}{\alpha_p\sigma_p}\right) + \frac{\underline{v}}{2\sigma_V}F_\gamma\left(\frac{\underline{v}y}{\alpha_p\sigma_p}\right) = \frac{\underline{v}(\underline{v}+\bar{v})}{2\sigma_V\bar{v}}F_\gamma\left(\frac{\underline{v}y}{\alpha_p\sigma_p}\right) = \frac{\underline{v}}{\sigma_V\bar{v}}\mu_V F_\gamma\left(\frac{\underline{v}y}{\alpha_p\sigma_p}\right) < 0,$$

which follows from $\underline{v}<0$.

□

Proof of Lemma A2: By the definition of expectation, G is equal to:

$$\begin{aligned} G &= \int_{-\sqrt{3}}^{\sqrt{3}}\int_{z\in A}\frac{z_v}{2\sqrt{3}}(\Delta\mu_X+\bar{\sigma}_1 z)F_\gamma\left(\frac{(\mu_V+\sigma_V z_v)(\Delta\mu_X+\bar{\sigma}z)}{\sigma_p}\right)f_Z(z)dzdz_v \\ &\quad + \int_{-\sqrt{3}}^{\sqrt{3}}\int_{z\in A}\frac{z_v}{2\sqrt{3}\sigma_p}(\mu_V+\sigma_V z_v)(\Delta\mu_X+\bar{\sigma}_1 z)(\Delta\mu_X+\bar{\sigma}z) \\ &\qquad f_\gamma\left(\frac{(\mu_V+\sigma_V z_v)(\Delta\mu_X+\bar{\sigma}z)}{\sigma_p}\right)f_Z(z)dzdz_v. \end{aligned} \tag{89}$$

Applying integration by parts to (89), we have:

$$\begin{aligned} G &= \int_{-\sqrt{3}}^{\sqrt{3}}\int_{z\in A}\frac{z_v}{2\sqrt{3}}(\Delta\mu_X+\bar{\sigma}_1 z)F_\gamma\left(\frac{(\mu_V+\sigma_V z_v)(\Delta\mu_X+\bar{\sigma}z)}{\sigma_p}\right)f_Z(z)dzdz_v \\ &\quad + \int_{z\in A}(\Delta\mu_X+\bar{\sigma}_1 z)\left[\frac{z_v(\mu_V+\sigma_V z_v)}{2\sqrt{3}\sigma_V}F_\gamma\left(\frac{(\mu_V+\sigma_V z_v)(\Delta\mu_X+\bar{\sigma}z)}{\sigma_p}\right)\right]_{z_v=-\sqrt{3}}^{z_v=\sqrt{3}}f_Z(z)dz \\ &\quad - \int_{-\sqrt{3}}^{\sqrt{3}}\int_{z\in A}\frac{z_v}{2\sqrt{3}}(\Delta\mu_X+\bar{\sigma}_1 z)F_\gamma\left(\frac{(\mu_V+\sigma_V z_v)(\Delta\mu_X+\bar{\sigma}z)}{\sigma_p}\right)f_Z(z)dzdz_v \\ &\quad - \int_{-\sqrt{3}}^{\sqrt{3}}\int_{z\in A}\frac{\mu_V+\sigma_V z_v}{2\sqrt{3}\sigma_V}(\Delta\mu_X+\bar{\sigma}_1 z)F_\gamma\left(\frac{(\mu_V+\sigma_V z_v)(\Delta\mu_X+\bar{\sigma}z)}{\sigma_p}\right)f_Z(z)dzdz_v \\ &= \int_{z\in A}\frac{\Delta\mu_X+\bar{\sigma}_1 z}{2\sigma_V}\left[\bar{v}F_\gamma\left(\frac{\bar{v}(\Delta\mu_X+\bar{\sigma}z)}{\sigma_p}\right)+\underline{v}F_\gamma\left(\frac{\underline{v}(\Delta\mu_X+\bar{\sigma}z)}{\sigma_p}\right)\right]f_Z(z)dz \end{aligned}$$

$$- \int_{-\sqrt{3}}^{\sqrt{3}} \int_{z\in A} \frac{\mu_V + \sigma_V z_v}{2\sqrt{3}\sigma_V} (\Delta\mu_X + \bar{\sigma}_1 z) F_\gamma \left(\frac{(\mu_V + \sigma_V z_v)(\Delta\mu_X + \bar{\sigma} z)}{\sigma_p} \right) f_Z(z) dz dz_v. \tag{90}$$

Now, consider the second part in the expression (90):

$$\int_{-\sqrt{3}}^{\sqrt{3}} \int_{z\in A} \frac{\mu_V + \sigma_V z_v}{2\sqrt{3}\sigma_V} (\Delta\mu_X + \bar{\sigma}_1 z) F_\gamma \left(\frac{(\mu_V + \sigma_V z_v)(\Delta\mu_X + \bar{\sigma} z)}{\sigma_p} \right) f_Z(z) dz dz_v. \tag{91}$$

Changing the variable z_v with $\frac{v_k - \mu_V}{\sigma_V}$ and using $\underline{v} = \mu_V - \sqrt{3}\sigma_V$ and $\bar{v} = \mu_V + \sqrt{3}\sigma_V$, the integral in (91) is equivalent to:

$$\int_{\underline{v}}^{\bar{v}} \int_{z\in A} \frac{v_k}{2\sqrt{3}\sigma_V^2} (\Delta\mu_X + \bar{\sigma}_1 z) F_\gamma \left(\frac{v_k(\Delta\mu_X + \bar{\sigma} z)}{\sigma_p} \right) f_Z(z) dz dv_k. \tag{92}$$

Applying integration by parts to (92), we obtain the following equality:

$$\begin{aligned} &\int_{\underline{v}}^{\bar{v}} \int_{z\in A} \frac{v_k}{2\sqrt{3}\sigma_V^2} (\Delta\mu_X + \bar{\sigma}_1 z) F_\gamma \left(\frac{v_k(\Delta\mu_X + \bar{\sigma} z)}{\sigma_p} \right) f_Z(z) dz dv_k \\ &= \int_{z\in A} \frac{\Delta\mu_X + \bar{\sigma}_1 z}{4\sqrt{3}\sigma_V^2} \left[\bar{v}^2 F_\gamma \left(\frac{\bar{v}(\Delta\mu_X + \bar{\sigma} z)}{\sigma_p} \right) - \underline{v}^2 F_\gamma \left(\frac{\underline{v}(\Delta\mu_X + \bar{\sigma} z)}{\sigma_p} \right) \right] f_Z(z) dz \\ &\quad - \int_{\underline{v}}^{\bar{v}} \int_{z\in A} \frac{v_k^2}{4\sqrt{3}\sigma_V^2 \sigma_p} (\Delta\mu_X + \bar{\sigma}_1 z)(\Delta\mu_X + \bar{\sigma} z) f_\gamma \left(\frac{v_k(\Delta\mu_X + \bar{\sigma} z)}{\sigma_p} \right) f_Z(z) dz dv_k. \end{aligned} \tag{93}$$

Using the condition (71) in (93), we have the following inequality:

$$\begin{aligned} &\int_{\underline{v}}^{\bar{v}} \int_{z\in A} \frac{v_k}{2\sqrt{3}\sigma_V^2} (\Delta\mu_X + \bar{\sigma}_1 z) F_\gamma \left(\frac{v_k(\Delta\mu_X + \bar{\sigma} z)}{\sigma_p} \right) f_Z(z) dz dv_k \\ &\le \int_{z\in A} \frac{\Delta\mu_X + \bar{\sigma}_1 z}{4\sqrt{3}\sigma_V^2} \left[\bar{v}^2 F_\gamma \left(\frac{\bar{v}(\Delta\mu_X + \bar{\sigma} z)}{\sigma_p} \right) - \underline{v}^2 F_\gamma \left(\frac{\underline{v}(\Delta\mu_X + \bar{\sigma} z)}{\sigma_p} \right) \right] f_Z(z) dz. \end{aligned} \tag{94}$$

By the equivalence of (91) and (92), (94) implies:

$$\begin{aligned} &\int_{-\sqrt{3}}^{\sqrt{3}} \int_{z\in A} \frac{\mu_V + \sigma_V z_v}{2\sqrt{3}\sigma_V} (\Delta\mu_X + \bar{\sigma}_1 z) F_\gamma \left(\frac{(\mu_V + \sigma_V z_v)(\Delta\mu_X + \bar{\sigma} z)}{\sigma_p} \right) f_Z(z) dz dz_v \\ &\le \int_{z\in A} \frac{\Delta\mu_X + \bar{\sigma}_1 z}{4\sqrt{3}\sigma_V^2} \left[\bar{v}^2 F_\gamma \left(\frac{\bar{v}(\Delta\mu_X + \bar{\sigma} z)}{\sigma_p} \right) - \underline{v}^2 F_\gamma \left(\frac{\underline{v}(\Delta\mu_X + \bar{\sigma} z)}{\sigma_p} \right) \right] f_Z(z) dz \\ &\Rightarrow \frac{1}{2\sigma_V} \int_{z\in A} (\Delta\mu_X + \bar{\sigma}_1 z) \bar{v} F_\gamma \left(\frac{\bar{v}(\Delta\mu_X + \bar{\sigma} z)}{\sigma_p} \right) f_Z(z) dz \\ &\quad \ge \frac{1}{2\sigma_V} \int_{z\in A} (\Delta\mu_X + \bar{\sigma}_1 z) \frac{\underline{v}^2}{\bar{v}} F_\gamma \left(\frac{\underline{v}(\Delta\mu_X + \bar{\sigma} z)}{\sigma_p} \right) f_Z(z) dz \\ &\qquad + \frac{2\sqrt{3}}{\bar{v}} \int_{-\sqrt{3}}^{\sqrt{3}} \int_{z\in A} \frac{\mu_V + \sigma_V z_v}{2\sqrt{3}} (\Delta\mu_X + \bar{\sigma}_1 z) F_\gamma \left(\frac{(\mu_V + \sigma_V z_v)(\Delta\mu_X + \bar{\sigma} z)}{\sigma_p} \right) f_Z(z) dz dz_v. \end{aligned} \tag{95}$$

Using (95) in (90), the following inequality could be written for G:

$$\begin{aligned} G \ge &\frac{1}{2\sigma_V} \int_{z\in A} (\Delta\mu_X + \bar{\sigma}_1 z) \frac{\underline{v}^2}{\bar{v}} F_\gamma \left(\frac{\underline{v}(\Delta\mu_X + \bar{\sigma} z)}{\sigma_p} \right) f_Z(z) dz \\ &+ \frac{2\sqrt{3}}{\bar{v}} \int_{-\sqrt{3}}^{\sqrt{3}} \int_{z\in A} \frac{\mu_V + \sigma_V z_v}{2\sqrt{3}} (\Delta\mu_X + \bar{\sigma}_1 z) F_\gamma \left(\frac{(\mu_V + \sigma_V z_v)(\Delta\mu_X + \bar{\sigma} z)}{\sigma_p} \right) f_Z(z) dz dz_v \end{aligned}$$

$$
\begin{aligned}
&+\frac{1}{2\sigma_V}\int_{z\in A}(\Delta\mu_X+\bar{\sigma}_1 z)\underline{v}F_\gamma\left(\frac{\underline{v}(\Delta\mu_X+\bar{\sigma}z)}{\sigma_p}\right)f_Z(z)dz\\
&-\int_{-\sqrt{3}}^{\sqrt{3}}\int_{z\in A}\frac{\mu_V+\sigma_V z_v}{2\sqrt{3}\sigma_V}(\Delta\mu_X+\bar{\sigma}_1 z)F_\gamma\left(\frac{(\mu_V+\sigma_V z_v)(\Delta\mu_X+\bar{\sigma}z)}{\sigma_p}\right)f_Z(z)dzdz_v\\
=&\frac{1}{2\sigma_V\bar{v}}\Big(\underline{v}\int_{z\in A}(\Delta\mu_X+\bar{\sigma}_1 z)(\bar{v}+\underline{v})F_\gamma\Big(\frac{\underline{v}(\Delta\mu_X+\bar{\sigma}z)}{\sigma_p}\Big)f_Z(z)dz\\
&+(4\sqrt{3}\sigma_V-2\bar{v})\int_{-\sqrt{3}}^{\sqrt{3}}\int_{z\in A}\frac{\mu_V+\sigma_V z_v}{2\sqrt{3}}(\Delta\mu_X+\bar{\sigma}_1 z)\\
&\qquad F_\gamma\Big(\frac{(\mu_V+\sigma_V z_v)(\Delta\mu_X+\bar{\sigma}z)}{\sigma_p}\Big)f_Z(z)dzdz_v\Big)\\
=&\frac{\underline{v}}{2\sigma_V\bar{v}}\Big(2\mu_V\int_{z\in A}(\Delta\mu_X+\bar{\sigma}_1 z)F_\gamma\Big(\frac{\underline{v}(\Delta\mu_X+\bar{\sigma}z)}{\sigma_p}\Big)f_Z(z)dz\\
&-\frac{1}{\sqrt{3}}\int_{-\sqrt{3}}^{\sqrt{3}}\int_{z\in A}(\mu_V+\sigma_V z_v)(\Delta\mu_X+\bar{\sigma}_1 z)F_\gamma\left(\frac{(\mu_V+\sigma_V z_v)(\Delta\mu_X+\bar{\sigma}z)}{\sigma_p}\right)f_Z(z)dzdz_v\Big)\\
=&\frac{\underline{v}}{2\sigma_V\bar{v}}\Big(2\mu_V\int_{z\in A}(\Delta\mu_X+\bar{\sigma}_1 z)F_\gamma\Big(\frac{\underline{v}(\Delta\mu_X+\bar{\sigma}z)}{\sigma_p}\Big)f_Z(z)dz\\
&-\frac{\mu_V}{\sqrt{3}}\int_{-\sqrt{3}}^{\sqrt{3}}\int_{z\in A}(\Delta\mu_X+\bar{\sigma}_1 z)F_\gamma\Big(\frac{(\mu_V+\sigma_V z_v)(\Delta\mu_X+\bar{\sigma}z)}{\sigma_p}\Big)f_Z(z)dzdz_v\\
&-\frac{\sigma_V}{\sqrt{3}}\int_{-\sqrt{3}}^{\sqrt{3}}\int_{z\in A}z_v(\Delta\mu_X+\bar{\sigma}_1 z)F_\gamma\Big(\frac{(\mu_V+\sigma_V z_v)(\Delta\mu_X+\bar{\sigma}z)}{\sigma_p}\Big)f_Z(z)dzdz_v\Big)\\
\geq&\, 0,
\end{aligned}
$$

where the last inequality follows from $\underline{v}<0$, Claim 1 and Claim 2, stated and proved at the end of this lemma.

Claim 1: Under conditions stated in Lemma A2, the following inequality holds:

$$\mathbb{E}_{Z_v,Z}\left[Z_v(\Delta\mu_X+\bar{\sigma}_1 Z)F_\gamma\left(\frac{(\mu_V+\sigma_V Z_v)(\Delta\mu_X+\bar{\sigma}Z)}{\sigma_p}\right)\mathcal{I}\{Z\in A\}\right]\geq 0.$$

Proof of Claim 1: First, note that $\mathbb{E}_Z\left[(\Delta\mu_X+\bar{\sigma}_1 Z)F_\gamma\left(\frac{(\mu_V+\sigma_V Z_v)(\Delta\mu_X+\bar{\sigma}Z)}{\sigma_p}\right)\mathcal{I}\{Z\in A\}\mid Z_v\right]$ is a random variable which is an increasing function of the random variable Z_v by the following equation:

$$
\begin{aligned}
&\frac{d}{dZ_v}\mathbb{E}_Z\left[(\Delta\mu_X+\bar{\sigma}_1 Z)F_\gamma\left(\frac{(\mu_V+\sigma_V Z_v)(\Delta\mu_X+\bar{\sigma}Z)}{\sigma_p}\right)\mathcal{I}\{Z\in A\}\mid Z_v\right]\\
&=\frac{\sigma_V}{\sigma_p}\mathbb{E}_Z\left[(\Delta\mu_X+\bar{\sigma}_1 Z)(\Delta\mu_X+\bar{\sigma}Z)f_\gamma\left(\frac{(\mu_V+\sigma_V Z_v)(\Delta\mu_X+\bar{\sigma}Z)}{\sigma_p}\right)\mathcal{I}\{Z\in A\}\mid Z_v\right]\\
&\geq 0,
\end{aligned}
\tag{96}
$$

where the last inequality follows from (71). Then, we have:

$$
\begin{aligned}
&\mathbb{E}_{Z_v,Z}\left[Z_v(\Delta\mu_X+\bar{\sigma}_1 Z)F_\gamma\left(\frac{(\mu_V+\sigma_V Z_v)(\Delta\mu_X+\bar{\sigma}Z)}{\sigma_p}\right)\mathcal{I}\{Z\in A\}\right]\\
&\quad=\mathbb{E}_{Z_v}\left[\mathbb{E}_Z\left[Z_v(\Delta\mu_X+\bar{\sigma}_1 Z)F_\gamma\left(\frac{(\mu_V+\sigma_V Z_v)(\Delta\mu_X+\bar{\sigma}Z)}{\sigma_p}\right)\mathcal{I}\{Z\in A\}\mid Z_v\right]\right]\\
&\quad=\mathbb{E}_{Z_v}\left[Z_v\mathbb{E}_Z\left[(\Delta\mu_X+\bar{\sigma}_1 Z)F_\gamma\left(\frac{(\mu_V+\sigma_V Z_v)(\Delta\mu_X+\bar{\sigma}Z)}{\sigma_p}\right)\mathcal{I}\{Z\in A\}\mid Z_v\right]\right]
\end{aligned}
$$

$$
\begin{aligned}
&= \mathbb{E}_{Z_v}[Z_v]\mathbb{E}_{Z_v,Z}\left[(\Delta\mu_X+\bar{\sigma}_1 Z)F_\gamma\left(\frac{(\mu_V+\sigma_V Z_v)(\Delta\mu_X+\bar{\sigma}Z)}{\sigma_p}\right)\mathcal{I}\{Z\in A\}\right]\\
&\qquad + \mathrm{Cov}\left(Z_v, \mathbb{E}_Z\left[(\Delta\mu_X+\bar{\sigma}_1 Z)F_\gamma\left(\frac{(\mu_V+\sigma_V Z_v)(\Delta\mu_X+\bar{\sigma}Z)}{\sigma_p}\right)\mathcal{I}\{Z\in A\}\mid Z_v\right]\right)\\
&= \mathrm{Cov}\left(Z_v, \mathbb{E}_Z\left[(\Delta\mu_X+\bar{\sigma}_1 Z)F_\gamma\left(\frac{(\mu_V+\sigma_V Z_v)(\Delta\mu_X+\bar{\sigma}Z)}{\sigma_p}\right)\mathcal{I}\{Z\in A\}\mid Z_v\right]\right)\\
&\geq 0,
\end{aligned}
$$

where the second equality follows from the law of iterated expectations, the forth equality follows from $\mathbb{E}_{Z_v}[Z_v]=0$ and the last inequality follows from (96).

Claim 2: Under conditions stated in Lemma A2, the following inequality holds:

$$
\begin{aligned}
&\int_{-\sqrt{3}}^{\sqrt{3}}\int_{z\in A}(\Delta\mu_X+\bar{\sigma}_1 z)F_\gamma\left(\frac{(\mu_V+\sigma_V z_v)(\Delta\mu_X+\bar{\sigma}z)}{\sigma_p}\right)f_Z(z)dzdz_v\\
&\qquad \geq 2\sqrt{3}\int_{z\in A}(\Delta\mu_X+\bar{\sigma}_1 z)F_\gamma\left(\frac{\underline{v}(\Delta\mu_X+\bar{\sigma}z)}{\sigma_p}\right)f_Z(z)dz.
\end{aligned}
$$

Proof of Claim 2: By (71), $\int_{z\in A}(\Delta\mu_X+\bar{\sigma}_1 z)F_\gamma\left(\frac{(\mu_V+\sigma_V z_v)(\Delta\mu_X+\bar{\sigma}z)}{\sigma_p}\right)f_Z(z)dz$ increases in z_v. Thus, we have:

$$
\begin{aligned}
&\int_{z\in A}(\Delta\mu_X+\bar{\sigma}_1 z)F_\gamma\left(\frac{(\mu_V+\sigma_V z_v)(\Delta\mu_X+\bar{\sigma}z)}{\sigma_p}\right)f_Z(z)dz\\
&\qquad \geq \int_{z\in A}(\Delta\mu_X+\bar{\sigma}_1 z)F_\gamma\left(\frac{\underline{v}(\Delta\mu_X+\bar{\sigma}z)}{\sigma_p}\right)f_Z(z)dz.
\end{aligned}
$$

The rest follows from the algebra:

$$
\begin{aligned}
&\int_{-\sqrt{3}}^{\sqrt{3}}\int_{z\in A}(\Delta\mu_X+\bar{\sigma}_1 z)F_\gamma\left(\frac{(\mu_V+\sigma_V z_v)(\Delta\mu_X+\bar{\sigma}z)}{\sigma_p}\right)f_Z(z)dzdz_v\\
&\qquad \geq \int_{-\sqrt{3}}^{\sqrt{3}}\int_{z\in A}(\Delta\mu_X+\bar{\sigma}_1 z)F_\gamma\left(\frac{\underline{v}(\Delta\mu_X+\bar{\sigma}z)}{\sigma_p}\right)f_Z(z)dzdz_v\\
&\qquad = 2\sqrt{3}\int_{z\in A}(\Delta\mu_X+\bar{\sigma}_1 z)F_\gamma\left(\frac{\underline{v}(\Delta\mu_X+\bar{\sigma}z)}{\sigma_p}\right)f_Z(z)dz.
\end{aligned}
$$

□

Proof of Lemma A3: The LHS of (72) could be written as:

$$
\begin{aligned}
&\mathbb{E}_Z\Big[(\Delta\mu_X+\bar{\sigma}_1 Z)(\Delta\mu_X+\bar{\sigma}Z)f_\gamma\Big(\frac{V_k(\Delta\mu_X+\bar{\sigma}Z)}{\sigma_p}\Big)\mid V_k=v_k\Big]\\
&\qquad = \mathbb{E}_Z\Big[(\Delta\mu_X+\bar{\sigma}Z)^2 f_\gamma\Big(\frac{V_k(\Delta\mu_X+\bar{\sigma}Z)}{\sigma_p}\Big)\mid V_k=v_k\Big]\\
&\qquad\qquad - (\bar{\sigma}-\bar{\sigma}_1)\mathbb{E}_Z\Big[(\Delta\mu_X+\bar{\sigma}Z)Zf_\gamma\Big(\frac{V_k(\Delta\mu_X+\bar{\sigma}Z)}{\sigma_p}\Big)\mid V_k=v_k\Big] \qquad (97)\\
&\qquad = \Delta\mu_X\mathbb{E}_Z\Big[(\Delta\mu_X+\bar{\sigma}Z)f_\gamma\Big(\frac{V_k(\Delta\mu_X+\bar{\sigma}Z)}{\sigma_p}\Big)\mid V_k=v_k\Big]\\
&\qquad\qquad + \bar{\sigma}_1\mathbb{E}_Z\Big[(\Delta\mu_X+\bar{\sigma}Z)Zf_\gamma\Big(\frac{V_k(\Delta\mu_X+\bar{\sigma}Z)}{\sigma_p}\Big)\mid V_k=v_k\Big]. \qquad (98)
\end{aligned}
$$

Now, we have two cases:

Case I: $\mathbb{E}_Z\Big[(\Delta\mu_X+\bar{\sigma}Z)Zf_\gamma\Big(\frac{V_k(\Delta\mu_X+\bar{\sigma}Z)}{\sigma_p}\Big)\mid V_k=v_k\Big]\le 0$. In this case, it trivially follows from (97) that $\mathbb{E}_Z\Big[(\Delta\mu_X+\bar{\sigma}_1 Z)(\Delta\mu_X+\bar{\sigma}Z)f_\gamma\Big(\frac{V_k(\Delta\mu_X+\bar{\sigma}Z)}{\sigma_p}\Big)\mid V_k=v_k\Big]\ge 0$ holds.

Case II: $\mathbb{E}_Z\Big[(\Delta\mu_X+\bar{\sigma}Z)Zf_\gamma\Big(\frac{V_k(\Delta\mu_X+\bar{\sigma}Z)}{\sigma_p}\Big)\mid V_k=v_k\Big]>0$. In this case, the nonnegativity of $\mathbb{E}_Z\Big[(\Delta\mu_X+\bar{\sigma}_1 Z)(\Delta\mu_X+\bar{\sigma}Z)f_\gamma\Big(\frac{V_k(\Delta\mu_X+\bar{\sigma}Z)}{\sigma_p}\Big)\mid V_k=v_k\Big]$ directly follows from (98) and Claim 1 proven at the end of this lemma.

Claim 1: We have $\mathbb{E}_Z\Big[(\Delta\mu_X+\bar{\sigma}Z)f_\gamma\Big(\frac{V_k(\Delta\mu_X+\bar{\sigma}Z)}{\sigma_p}\Big)\mid V_k=v_k\Big]\le 0$ for any $v_k\in[\underline{v},\bar{v}]$.

Proof of Claim 1: By the definition of expectation and the independence of V_k and Z, the expression $\mathbb{E}_Z\Big[(\Delta\mu_X+\bar{\sigma}Z)f_\gamma\Big(\frac{V_k(\Delta\mu_X+\bar{\sigma}Z)}{\sigma_p}\Big)\mid V_k=v_k\Big]$ could be written as:

$$\mathbb{E}_Z\Big[(\Delta\mu_X+\bar{\sigma}Z)f_\gamma\Big(\frac{V_k(\Delta\mu_X+\bar{\sigma}Z)}{\sigma_p}\Big)\mid V_k=v_k\Big]=\int_{-\infty}^{\infty}(\Delta\mu_X+\bar{\sigma}z)f_\gamma\Big(\frac{v_k(\Delta\mu_X+\bar{\sigma}z)}{\sigma_p}\Big)f_Z(z)dz, \quad (99)$$

where $f_Z(z)$ denotes the pdf of a standard normal random variable at point z. Replacing z with $\frac{x_d-\Delta\mu_X}{\bar{\sigma}}$ in the RHS of (99), we obtain:

$$\mathbb{E}_Z\Big[(\Delta\mu_X+\bar{\sigma}Z)f_\gamma\Big(\frac{V_k(\Delta\mu_X+\bar{\sigma}Z)}{\sigma_p}\Big)\mid V_k=v_k\Big]=\int_{-\infty}^{\infty}x_d f_\gamma\Big(\frac{v_k x_d}{\sigma_p}\Big)f_X(x_d;\Delta\mu_X,\bar{\sigma})dx_d, \quad (100)$$

where $f_X(x_d;\Delta\mu_X,\bar{\sigma})$ represents normal pdf with mean $\Delta\mu_X$ and standard deviation $\bar{\sigma}$ at point x_d. By algebra, we can arrange the RHS of (100) as follows:

$$\begin{aligned}
&\mathbb{E}_Z\Big[(\Delta\mu_X+\bar{\sigma}Z)f_\gamma\Big(\frac{V_k(\Delta\mu_X+\bar{\sigma}Z)}{\sigma_p}\Big)\mid V_k=v_k\Big]\\
&=\int_{-\infty}^{0}x_d f_\gamma\Big(\frac{v_k x_d}{\sigma_p}\Big)f_X(x_d;\Delta\mu_X,\bar{\sigma})dx_d+\int_0^{\infty}x_d f_\gamma\Big(\frac{v_k x_d}{\sigma_p}\Big)f_X(x_d;\Delta\mu_X,\bar{\sigma})dx_d\\
&=-\int_0^{\infty}x_d f_\gamma\Big(\frac{v_k(-x_d)}{\sigma_p}\Big)f_X(-x_d;\Delta\mu_X,\bar{\sigma})dx_d+\int_0^{\infty}x_d f_\gamma\Big(\frac{v_k x_d}{\sigma_p}\Big)f_X(x_d;\Delta\mu_X,\bar{\sigma})dx_d\\
&=\int_0^{\infty}x_d f_\gamma\Big(\frac{v_k x_d}{\sigma_p}\Big)\left(f_X(x_d;\Delta\mu_X,\bar{\sigma})-f_X(-x_d;\Delta\mu_X,\bar{\sigma})\right)dx_d\\
&=\int_0^{\infty}x_d f_\gamma\Big(\frac{v_k x_d}{\sigma_p}\Big)\left(\frac{1}{\sqrt{2\pi}\bar{\sigma}}\mathrm{e}^{-\frac{(x_d-\Delta\mu_X)^2}{2\bar{\sigma}^2}}-\frac{1}{\sqrt{2\pi}\bar{\sigma}}\mathrm{e}^{-\frac{(-x_d-\Delta\mu_X)^2}{2\bar{\sigma}^2}}\right)dx_d\\
&=\frac{1}{\sqrt{2\pi}\bar{\sigma}}\int_0^{\infty}x_d f_\gamma\Big(\frac{v_k x_d}{\sigma_p}\Big)\mathrm{e}^{-\frac{x_d^2+\Delta\mu_X^2}{2\bar{\sigma}^2}}\left(\mathrm{e}^{\frac{\Delta\mu_X x_d}{\bar{\sigma}^2}}-\mathrm{e}^{-\frac{\Delta\mu_X x_d}{\bar{\sigma}^2}}\right)dx_d\\
&\le 0,
\end{aligned}$$

where the second equality follows from replacing x_d with $-x_d$ in the first integral, the third equality follows from the symmetry of the logistic distribution, the fourth equality follows from substituting the formula of the normal pdf for $f_X(x_d;\Delta\mu_X,\bar{\sigma})$. Finally, the last inequality follows from $\Delta\mu_X<0$ and $x_d\ge 0$.

□